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Two Graphs And Their Permutation Matrix: A Simple Solution To Isomorphic Problem of Kinematic Chains

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ABSTRACT: *Isomorphism is one of the most elusive properties of kinematic chains which attracted attention of many researchers. Its identification is an essential step during number synthesis in order to avoid duplicacy and time consumption. In this paper a simple but effective method, based on matrix theory, is used to identify the isomorphism. In the first step, graphs and the adjacency matrices of two kinematic chains are generated and then their permutation matrix is obtained by using an algorithm. This permutation matrix is then used to satisfy a theorem which, in turn, provides a base for identification of isomorphism. Single and multidegree of freedom kinematic chains with simple joints and up to 12 links are tested for the proposed method. The proposed test provides logical results to each group under consideration.*
Keywords: *Adjacency matrix, Graph, Isomorphism, Kinematic chain, Mechanism, Permutation matrix.*

I. INTRODUCTION

Every machine involves a mechanism and mechanism involves a specific kinematic chain. This chain contains a particular number of links, joints and degree of freedom. With a given number of links, joints and degree of freedom a number of kinematic chains can be created. This is called number synthesis and the property which decides similarity in structures of kinematic chains is called isomorphism. This must be identified at the initial stage, otherwise a lot of time and effort would be wasted. Many researchers have contributed in this area [1-15]. Read and Corneil [1] discussed about different methods available for isomorphism testing and pointed out some of the difficulties in isomorphism identification. Ding and Huang [2] discussed about the characteristic adjacency matrices for the identification of isomorphism. Yan and Chiu [3] presented a literature review on the number synthesis of kinematic chains of mechanisms. Rao and Varada Raju [4] discussed Hamming number technique for identification of isomorphism but secondary Hamming string is needed when primary Hamming string fails. Ambekar and Agrawal [5] proposed Canonical coding of kinematic chains but it becomes computationally uneconomical when applied to large kinematic chains. Chu and Cao [6] discussed about adjacent- chain table method to identify isomorphism. Ding and Huang [7] proposed relabeling the vertices of the perimeter graphs canonically when characteristic polynomial and Eigen value approach fails. Rao [8] discussed about Fuzzy numbers to investigate isomorphism among kinematic chains and inversions. Yan and Hwang [9] proposed a set of identification numbers for the identification of isomorphism of planar linkage chain with turning-pairs only. Zeng et al [10] proposed the Dividing and Matching Algorithm, based on first dividing all the vertices into groups and then matching them for isomorphism identification. Dobrjanskyj and Freudenstein [11] discussed about a computerized method based on graph theory for identification of isomorphism, and automatic sketching of the graph of mechanisms based on incidence matrices. Cubillo and Wan [12] discussed about the proof for necessary and sufficient conditions of the eigenvalues and eigenvectors of adjacent matrices for isomorphic kinematic chains. They also discussed about a new procedure to compare eigenvalues and several eigenvectors of adjacent matrices of isomorphic kinematic chains to identify the isomorphic chains. Chang et al [13] also proposed comparison of the eigenvalues and eigenvectors of adjacent matrices for isomorphism identification. Ding et al [14] proposed a method to automatically synthesize the complete set of planar non-fractionated kinematic chains with up to six independent loops discussed about an improved algorithm to obtain the simplified characteristic codes of topological graphs to increase the efficiency of isomorphism identification. Uicker and Raicu [15] discuss about the advantage of link-link form of incidence matrix over link joint form of incidence matrix. Many methods, as mentioned in literature, are available but most of them are very complicated and lengthy. Therefore, a simple, efficient and reliable method is required. This paper utilizes a simple concept of graph theory and claims to be a stronger tool in the isomorphism identification area.

II. TERMINOLOGY AND DEFINITIONS

2.1 Graph Of A Kinematic Chain

A graph G consists of a set V of vertices and a collection E (not necessarily a set) of unordered pairs of vertices called edges. A kinematic chain is usually represented by a graph. A link of a chain is represented by a vertex V and a joint of the chain is represented by an edge E in the graph. For example, the graph corresponding to the four-bar chain, Fig.1 (a) is shown in Fig.1 (b).

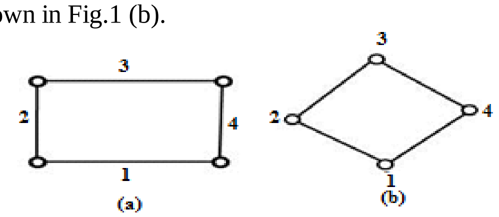


Fig.1: (a) Four-bar chain and (b) its graph representation

2.2 Permutation matrix

A permutation matrix is a square binary matrix that has exactly one 1 in each row and column and zero elsewhere. 5 × 5 matrix is taken for example only.

$$P = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \quad (1)$$

2.3 Identity matrix

It is a square binary matrix having principle diagonal elements one and zero elsewhere. 5 × 5 matrix is taken for example only.

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

2.4 Adjacency matrix

It is a square matrix representing the connectivity of other vertices (or connectivity of other links in terms of kinematic chain) with the vertex under consideration (or the link under consideration). When a vertex (link) is directly connected to other vertex (link) then 1 is provided in the matrix, otherwise 0. Connectivity of a vertex with itself is also taken zero.

2.5 Testing isomorphism

Two graphs are isomorphic if and only if their adjacency matrices are isomorphic. But two adjacency matrices A and A_1 are isomorphic if there is a permutation matrix P such that

$$A_1P = PA \quad (3)$$

This permutation matrix P depends on the isomorphism f of the two graphs.

III. DISCUSSION ABOUT THE METHOD

Suppose A and A_1 are the two adjacency matrices of two isomorphic graphs. Then one of these matrices can be obtained from the other by rearranging rows and rearranging the corresponding columns. Now rearranging rows of A is equivalent to premultiplying by a permutation matrix P obtaining the product matrix PA. The subsequent rearrangement of corresponding columns is equivalent to postmultiplying PA by P^{-1}. Thus $A_1 = PAP^{-1}$. Conversely, if $A_1P = PA$, A_1 can be obtained from A by rearranging columns and then rows, showing the two graphs are isomorphic.

3.1 Illustration 1

Consider two simple graphs G and G_1, as shown in Fig. 2.

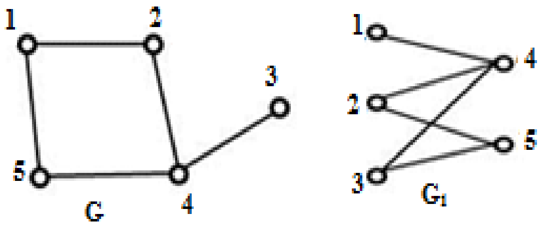


Fig. 2: Two simple graphs

The isomorphic relationships between vertices of graphs G and G_1 are given in Table 1.

Table 1: Isomorphic relationship between vertices of graph G and G_1

Isomorphism f of any vertex of graph G to any vertex of graph G_1		
Vertex of graph G with its degree in bracket	Similar vertices of graph G_1 with same degree	Selected isomorphic vertex of graph G_1
1 (2)	2, 3 and 5	5
2 (2)	2 and 3	2
3 (1)	1	1
4 (3)	4	4
5 (2)	3	3

From the Table 1, it is clear that every vertex of graph G has an isomorphic relation with every vertex of graph G_1. Therefore, as per our theory discussed in section 2.5, permutation matrix is possible for these two graphs. Therefore, the adjacency matrices of G and G_1 are:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \qquad (4)$$

$$A_1 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} \qquad (5)$$

Now replace fifth row of A by its first row, first row by its third row and third row by its fifth row. Second and fourth rows will not be changed. This is because there exists an isomorphic relationship of same vertex of graph G with the same vertex of graph G_1. This is given in Table 1. Suppose the resulting matrix is denoted by B.

$$B = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \qquad (6)$$

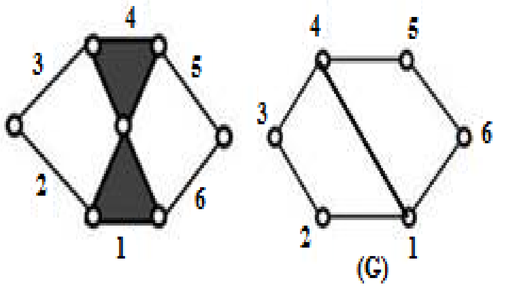
If the same sequences of operations are performed on an identity matrix of 5 × 5, then the following permutation matrix is obtained:

$$P = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (7)$$

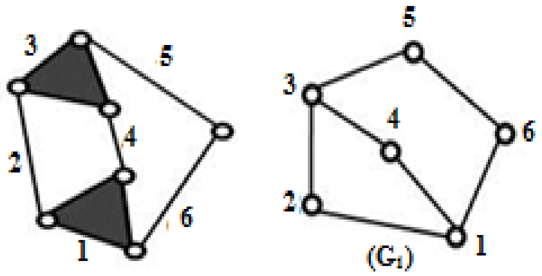
As per our theory discussed in section 2.5, PA = B = A_1P. This proves that the two graphs G and G_1 are isomorphic. This also proves that A_1 can be obtained from A if the same sequences of operations are performed on columns also.

3.2 Illustration 2

Consider Watt's chain and Stephenson's chain and their corresponding graphs G and G_1, as shown in Fig. 3.



Watt's chain and its graph



Stephenson's chain and its graph

Fig. 3: Six links kinematic chains and their graphs

The isomorphic relationships between vertices of graphs G and G_1 are given in Table 2.

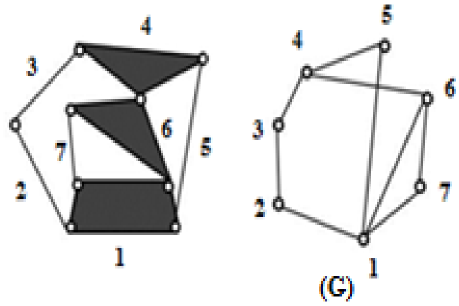
Table 2: Isomorphic relationship between vertices of graph G and G_1

Isomorphism f of any vertex of graph G to any vertex of graph G_1		
Vertex of graph G with its degree in bracket	Similar vertices of graph G_1 with same degree	Selected isomorphic vertex of graph G_1
1 (3)	1 and 3	None
2 (2)	2, 4, 5 and 6	5
3 (2)	2, 4 and 6	6
4 (3)	1 and 3	1
5 (2)	2 and 4	None
6 (2)	2 and 4	None

From the Table 2, it is clear that vertices 1, 5 and 6 of graph G are not isomorphic with the remaining vertices of graph G_1. Therefore, as per our theory discussed in section 2.5, permutation matrix is not possible for these two graphs and hence the two kinematic chains are declared non-isomorphic.

3.3 Illustration 3

Consider seven links chain and their corresponding graphs G and G_1, as shown in Fig. 4



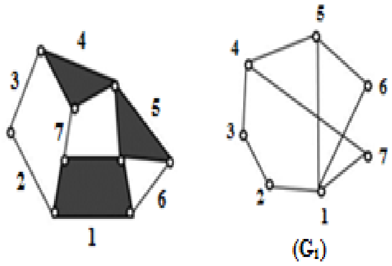


Fig. 4: Seven links kinematic chains and their graphs

The isomorphic relationships between vertices of graphs G and G_1 are given in Table 3.

Table 3: Isomorphic relationship between vertices of graph G and G_1

Isomorphism f of any vertex of graph G to any vertex of graph G_1		
Vertex of graph G with its degree in bracket	Similar vertices of graph G_1 with same degree	Selected isomorphic vertex of graph G_1
1 (4)	1	1
2 (2)	2, 3, 6 and 7	2
3 (2)	3, 6 and 7	3
4 (3)	4 and 5	4
5 (2)	6 and 7	6
6 (3)	5	5
7 (2)	7	7

From the Table 3, it is clear that every vertex of graph G has an isomorphic relation with every vertex of graph G_1. Therefore, as per our theory discussed in section 2.5, permutation matrix is possible for these two graphs. Therefore, the adjacency matrices of G and G_1 are

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad (8)$$

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \quad (9)$$

Now replace the sixth row of A by its fifth row and fifth row by its sixth row. Other rows will not be changed. This is because there exists an isomorphic relationship of same vertex of graph G with the same vertex of graph G_1. This is given in Table 3. Suppose the resulting matrix is denoted by B.

$$B = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad (10)$$

If same sequences of operations are performed on an identity matrix of 7×7, following permutation matrix is obtained:

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad (11)$$

As per our theory discussed in section 2.5, PA = B = A_1P. This proves that the two graphs G and G_1 are isomorphic. This also proves that A_1 can be obtained from A if the same sequences of operations are performed on columns also.

3.4 Illustration 4

Consider a pair of eight links kinematic chains and their graphs G and G_1, as shown in Fig. 5

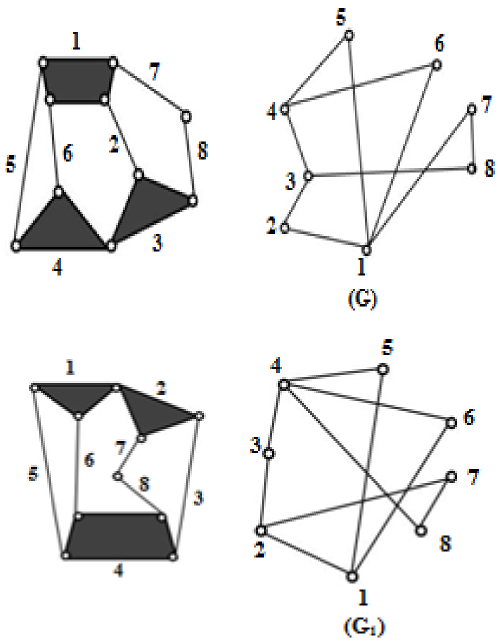


Fig. 5: Eight links kinematic chains and their graphs

The isomorphic relationships between vertices of graphs G and G_1 are given in Table 4.

Table 4: Isomorphic relationship between vertices of graph G and G_1

Isomorphism f of any vertex of graph G to any vertex of graph G_1		
Vertex of graph G with its degree in bracket	Similar vertices of graph G_1 with same degree	Selected isomorphic vertex of graph G_1
1 (4)	4	4
2 (2)	3, 5, 6, 7 and 8	3
3 (3)	1 and 2	2
4 (3)	1	1
5 (2)	5, 6, 7 and 8	5
6 (2)	6, 7 and 8	6
7 (2)	7 and 8	7
8 (2)	8	8

From the Table 4, it is clear that every vertex of graph G has an isomorphic relation with every vertex of graph G_1. Therefore, as per our theory discussed in section 2.5, permutation matrix is possible for these two graphs. The adjacency matrices of G and G_1 are:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad (12)$$

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix} \quad (13)$$

Now replace the fourth row of A by its first row, third row by its second row, second row by its third row, first row by its fourth row, eight row by its seventh row and seventh row by its eighth row. Rows fifth and sixth will not be changed. This is because there exists an isomorphic relationship of same vertex of graph G with the same vertex of graph G_1. This is given in Table 4. Suppose the resulting matrix is denoted by B.

$$B = \begin{bmatrix} 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (14)$$

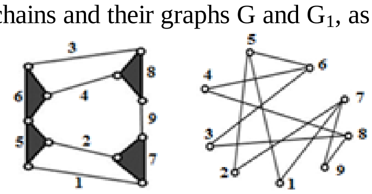
If same sequences of operations are performed on an identity matrix of 8 × 8, following permutation matrix is obtained:

$$P = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad (15)$$

As per our theory discussed in section 2.5, PA = B = A_1P. This proves that the two graphs G and G_1 are isomorphic. This also proves that A_1 can be obtained from A if the same sequences of operations are performed on columns also.

3.5 Illustration 5

Consider a pair of nine links kinematic chains and their graphs G and G_1, as shown in Fig. 6



(G)

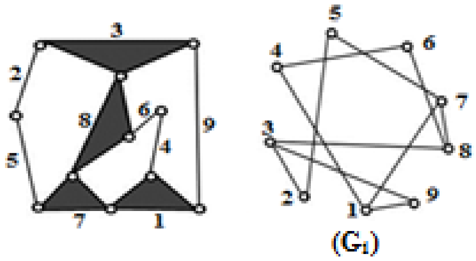


Fig. 6: Nine links kinematic chains and their graphs

The isomorphic relationships between vertices of graphs G and G_1 are given in Table 5.

Table 5: Isomorphic relationship between vertices of graph G and G_1

Isomorphism f of any vertex of graph G to any vertex of graph G_1		
Vertex of graph G with its degree in bracket	Similar vertices of graph G_1 with same degree	Selected isomorphic vertex of graph G_1
1 (2)	2, 4, 5, 6 and 9	9
2 (2)	2, 4, 5 and 6	None
3 (2)	2, 4, 5 and 6	None
4 (2)	2, 4, 5 and 6	None
5 (3)	1, 3, 7 and 8	1
6 (3)	3, 7 and 8	3
7 (3)	7 and 8	None
8 (3)	7 and 8	None
9 (2)	2, 4, 5 and 6	None

From the Table 5, it is clear that vertices 2, 3, 4, 7, 8 and 9 of the graph G are not isomorphic with the remaining vertices of the graph G_1. Therefore, as per our theory discussed in section 2.5, permutation matrix is not possible for these two graphs and hence the two kinematic chains are declared non-isomorphic.

3.6 Illustration 6

Consider a pair of ten links kinematic chains and their graphs G and G_1, as shown in Fig. 7

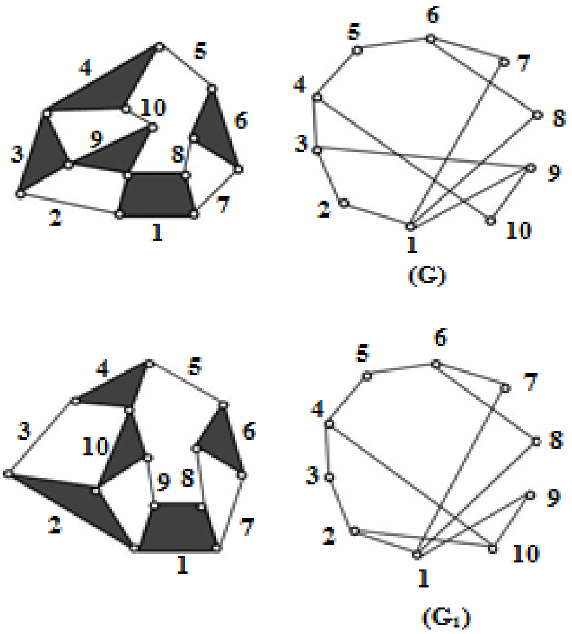


Fig. 7: Ten links kinematic chains and their graphs

The isomorphic relationships between vertices of graphs G and G_1 are given in Table 6.

Table 6: Isomorphic relationship between vertices of graph G and G_1

Isomorphism f of any vertex of graph G to any vertex of graph G_1		
Vertex of graph G with its degree in bracket	Similar vertices of graph G_1 with same degree	Selected isomorphic vertex of graph G_1
1 (4)	1	1
2 (2)	3, 5, 7, 8 and 9	7
3 (3)	2, 4, 6 and 10	10
4 (3)	2, 4 and 6	4
5 (2)	3, 5, 8 and 9	5
6 (3)	2 and 6	6
7 (2)	3, 8 and 9	8
8 (2)	3 and 9	9
9 (3)	2	2
10 (2)	3	3

From the Table 6, it is clear that every vertex of graph G has an isomorphic relation with every vertex of graph G_1. Therefore, as per our theory discussed in section 2.5, permutation matrix is possible for these two graphs. The adjacency matrices of G and G_1 are:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad (16)$$

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad (17)$$

Now replace the seventh row of A by its second row, tenth row by its third row, eight row by its seventh row, ninth row by its eight row, second row by its ninth row and third row by its tenth row. Rows first, fourth, fifth, sixth will not be changed. This is because there exists an isomorphic relationship of same vertex of graph G with the same vertex of graph G_1. This is given in Table 6. Suppose the resulting matrix is denoted by B.

$$B = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad (18)$$

If same sequences of operations are performed on an identity matrix of 10×10, following permutation matrix is obtained:

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (19)$$

As per our theory discussed in section 2.4, PA = B = A_1P. This proves that the two graphs G and G_1 are isomorphic. This also proves that A_1 can be obtained from A if the same sequences of operations are performed on columns also.

3.7 Illustration 7

Consider a pair of eleven links kinematic chains and their graphs G and G_1, as shown in Fig. 8

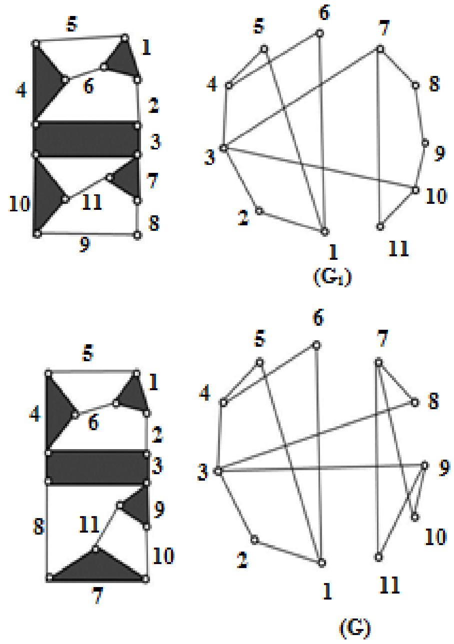


Fig. 8: Eleven links kinematic chains and their graphs

The isomorphic relationships between vertices of graphs G and G_1 are given in Table 7. From the Table 7, it is clear that vertices 3, 7, 8 and 11 of the graph G are not isomorphic with the remaining vertices of the graph G_1. Therefore, as per our theory discussed in section 2.5, permutation matrix is not possible for these two graphs and hence the two kinematic chains are declared non-isomorphic.

Table 7: Isomorphic relationship between vertices of graph G and G_1

Isomorphism f of any vertex of graph G to any vertex of graph G_1		
Vertex of graph G with its degree in bracket	Similar vertices of graph G_1 with same degree	Selected isomorphic vertex of graph G_1
1 (3)	1, 4, 7 and 10	1
2 (2)	2, 5, 6, 8, 9 and 11	2
3 (4)	3	None
4 (3)	4, 7 and 10	4
5 (2)	5, 6, 8, 9 and 11	5
6 (2)	6, 8, 9 and 11	6
7 (3)	7 and 10	None
8 (2)	8, 9 and 11	None
9 (3)	7 and 10	7
10 (2)	8, 9 and 11	11
11(2)	8 and 9	None

3.8 Illustration 8

Consider a pair of twelve links kinematic chains and their graphs G and G_1, as shown in Fig. 9

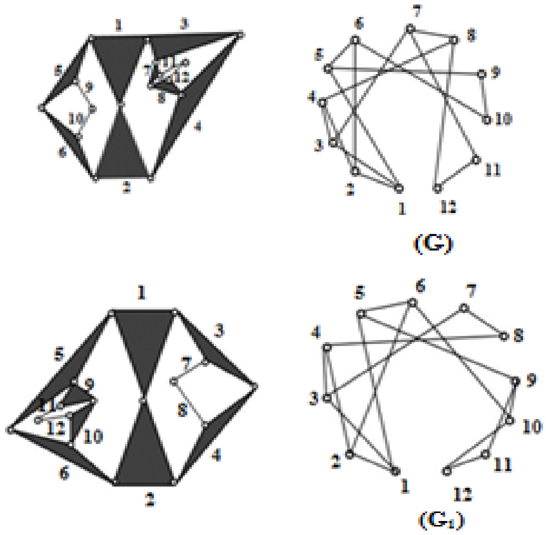


Fig. 9: Twelve links kinematic chains and their graphs

The isomorphic relationships between vertices of graphs G and G_1 are given in Table 8.

Table 8: Isomorphic relationship between vertices of graph G and G_1

Isomorphism f of any vertex of graph G to any vertex of graph G_1		
Vertex of graph G with its degree in bracket	Similar vertices of graph G_1 with same degree	Selected isomorphic vertex of graph G_1
1 (3)	1, 2, 3, 4, 5, 6, 9 and 10	1
2 (3)	2, 3, 4, 5, 6, 9 and 10	2
3 (3)	3, 4, 5, 6, 9 and 10	5
4 (3)	3, 4, 6, 9 and 10	6
5 (3)	3, 4, 9 and 10	3
6 (3)	4, 9 and 10	4
7 (3)	9 and 10	9
8 (3)	10	10
9 (2)	7, 8, 11 and 12	7
10 (2)	8, 11 and 12	8
11 (2)	11 and 12	11
12 (2)	12	12

From the Table 8, it is clear that every vertex of graph G has an isomorphic relation with every vertex of graph G_1. Therefore, as per our theory discussed in section 2.5, permutation matrix is possible for these two graphs. The adjacency matrices of G and G_1 are:

$$A = \begin{bmatrix} 0&1&1&0&1&0&0&0&0&0&0&0 \\ 1&0&0&1&0&1&0&0&0&0&0&0 \\ 1&0&0&1&0&0&1&0&0&0&0&0 \\ 0&1&1&0&0&0&0&1&0&0&0&0 \\ 1&0&0&0&0&1&0&0&1&0&0&0 \\ 0&1&0&0&1&0&0&0&0&1&0&0 \\ 0&0&1&0&0&0&0&1&0&0&1&0 \\ 0&0&0&1&0&0&1&0&0&0&0&1 \\ 0&0&0&0&1&0&0&0&0&1&0&0 \\ 0&0&0&0&0&1&0&0&1&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&1 \\ 0&0&0&0&0&0&0&1&0&0&1&0 \end{bmatrix} \quad (20)$$

$$A_1 = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix} \quad (21)$$

Now replace the fifth row of A by its third row, sixth row by its fourth row, third row by its fifth row, fourth row by its sixth row, ninth row by its seventh row, tenth row by its eight row, seventh row by its ninth row and eighth row by its tenth row. Rows first, second, eleventh and twelfth will not be changed. This is because there exists an isomorphic relationship of same vertex of graph G with the same vertex of graph G_1. This is given in Table 8. Suppose the resulting matrix is denoted by B.

$$B = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix} \quad (22)$$

If same sequences of operations are performed on an identity matrix of 12 × 12, following permutation matrix is obtained:

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (23)$$

As per our theory discussed in section 2.5, PA = B = A_1P. This proves that the two graphs G and G_1 are isomorphic. This also proves that A_1 can be obtained from A if the same sequences of operations are performed on columns also.

4. Conclusion

A simple but effective method, called permutation matrix method, is used to identify the isomorphism of single and multidegree of freedom kinematic chains with simple joints and up to 12 links. The results, as per our different illustrations, show its effectiveness in identifying isomorphism. Results and method used in this work would be beneficial to the automation of isomorphism identification of the kinematic chains.

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