

# Numerical Modeling of Aircraft Wing Deflection under Different Loading Conditions

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**ABSTRACT:** In engineering designs such as in buildings, bridges, automobiles, aircraft bodies, points of failure are of critical importance to the designers and researchers. The course of this failure has always been attributed to the structural engineering components being subjected to different stresses and loading systems. This paper analyses deflection pattern in aircraft wing under various loading systems. Two-dimensional standard deflection (flexural) equation was used in the modeling of a typical aircraft wing similar to that of an Airbus A320 wing. Finite element method, one of the numerical methods was adopted in the model development which was based on the classical plate theory and plane stress assumptions. The model was then used to carry out analysis on the wing which was taken as a thin plate. The wing was discretized with triangular and rectangular meshing. JAVA code was utilised in solving the model algorithm which was also made to be interactive. The model analysed different cases with different boundary conditions and loading conditions. Two cases were analysed and deflection along X, Y and XY-axis for each case were generated. Results obtained established that the greater the load, the higher the deflection obtained in the aircraft wing under different loading conditions. However, for edges and nodes with fixed and roller support, higher loadings is required for deflection to occur

**KEYWORDS:** Aircraft Wing, Plate theory, Finite Element Analysis, Loadings, Deflection

Date of Submission: 16-01-2019

Date of acceptance: 28-01-2019

## I. INTRODUCTION

Structural components in engineering such as buildings, bridges, automobiles, aircraft bodies etc. are subjected to different stresses and loading systems. These stresses cause structural failure and deformation consequently in these components therefore structural analysis before their construction is the most delicate and of course a pre – requisite operation that must be carried out. The aircraft wings are the primary lift producing device for an aircraft. The aircraft wings are designed aerodynamically to generate lift force which is required in order for an aircraft to fly. Besides generating the necessary lift force, the aircraft wings are used to carry the fuel required for the mission by the aircraft, can have mounted engines or can carry extra fuel tanks or other armaments. The basic goal of the wing is to generate lift and minimize drag as far as possible. When the airflow passes the wing at any suitable angle of attack, a pressure differential is created. A region of lower pressure is created over the top surface of the wing while a region of higher pressure is created below the surface of the wing (Pungoti et al., 2014).

It is important that all the different types of loads that the aircraft wing will bear be well estimated, and then the structural response to those loads be carefully calculated. To carefully calculate the response of the aircraft wing to estimated or measured loadings, it is important to use structural analysis techniques to which considerable confidence can be assigned (Bruce K. Donaldson, 2008). Of all the numerical methods for such complex analysis, finite element method demonstrates greater potential for such analysis. The use of finite element analysis in structural analysis have been increasing over the years as it has proven an alternative to mathematical analysis mostly giving a close approximation. The finite element technique of plane stress analysis has been presented in different papers. The technique has gained considerable recognition with application to problems associated with the aircraft industry. In all of these papers, however, the technique has been applied to problems associated with isotropic materials. A. C. Maki (1968) examined the framework method and the stiffness element method closely and determined their applicability in handling problems in orthotropic plane

stress. RuslizamDaud et al. (2012) introduced the concept of fracture mechanics and numerical approach to solve interacting cracks problems in solid bodies which involves elastic crack interaction. A new computational fracture mechanics algorithm was developed by adopting stress singularity approach in finite element (FE) formulation. As a conclusion, the FE formulated approach was found to be at agreeable accuracy with analytical formulation. Some other researchers have also worked on plate bending analysis using finite element method but used softwares with finite element working environment. This involves first getting the mathematical model of such problems and using the software to model the problem and a keen comparison will be made to highlight the best procedure although, the programming model always proved more efficient. M. Zdiri et al (2009) developed the design of rigid RCC (Roller Compacted Concrete) pavements gradually through methods established by various organisms for the determination of the necessary thicknesses of roadways. In the study, a numerical 3D modelling was used by introducing to the computer code "Abaqus" the behaviour law of the RCC. Finally, the results of 3D modelling were compared with those obtained by the various other methods. The comparison shows good correspondences although the 3D modelling gives results slightly lower than those given by the 2D methods in stresses. F. Carta and A. Pirondi (2011) also worked on a study to validate a numerical method of analysis that can predict the damage tolerance of these reinforced panels. Therefore, using a fracture mechanics approach, several models (different by the geometry and the types of reinforcement constraints) were simulated with the finite element solver ABAQUS. The stress intensity factor trend obtained numerically as a function of crack growth was used to determine the fatigue crack growth rate, obtaining a good approximation of the experimental crack propagation rate in the skin. This paper aims at presenting the analysis of deflection of an aircraft wing under different loading and boundary conditions using finite element method. The finite element analysis was done and was written in JAVA code as an interactive computer model.

## II. MATHEMATICAL MODEL

### Governing Equations

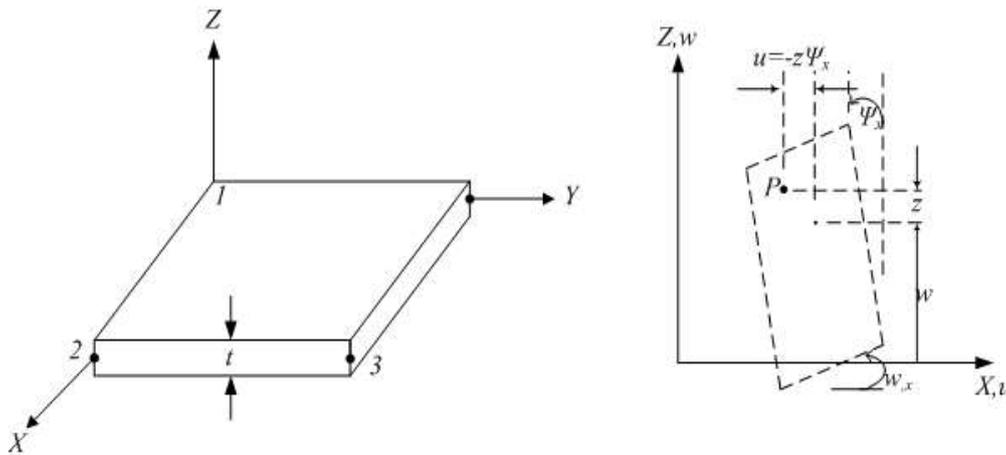


Fig.1 Thin Plate Element

Upon bending, particles that were on the mid-surface  $z = 0$  undergo a deflection  $w(x, y)$  along  $z$ . The slopes of the mid-surface in the  $x$  and  $y$  directions are  $\partial w/\partial x$  and  $\partial w/\partial y$ . The rotations of the material normal about  $x$  and  $y$  are denoted by  $\theta_x$  and  $\theta_y$ , respectively. For small deflections and rotations the foregoing kinematic assumption relates these rotations to the slopes:

$$\theta_x = \frac{\partial w}{\partial y}, \quad \theta_y = -\frac{\partial w}{\partial x}.$$

The displacements  $\{u_x, u_y, u_z\}$  of a plate particle  $P(x, y, z)$  not necessarily located on the mid-surface are given by

$$u_x = -z \frac{\partial w}{\partial x} = z\theta_y, \quad u_y = -z \frac{\partial w}{\partial y} = -z\theta_x, \quad u_z = w.$$

Where,  $w$  is the deflection of the middle plane of the plate in the  $z$  direction. Further the relationship between, the strain and deflection is given by,

$$\begin{aligned}
 e_{xx} &= \frac{\partial u_x}{\partial x} = -z \frac{\partial^2 w}{\partial x^2} = -z \kappa_{xx}, \\
 e_{yy} &= \frac{\partial u_y}{\partial y} = -z \frac{\partial^2 w}{\partial y^2} = -z \kappa_{yy}, \\
 e_{zz} &= \frac{\partial u_z}{\partial z} = -z \frac{\partial^2 w}{\partial z^2} = 0, \\
 2e_{xy} &= \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} = -2z \frac{\partial^2 w}{\partial x \partial y} = -2z \kappa_{xy}, \\
 2e_{xz} &= \frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x} = -\frac{\partial w}{\partial x} + \frac{\partial w}{\partial x} = 0, \\
 2e_{yz} &= \frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y} = -\frac{\partial w}{\partial y} + \frac{\partial w}{\partial y} = 0.
 \end{aligned}$$

Here,

$$\kappa_{xx} = \frac{\partial^2 w}{\partial x^2}, \quad \kappa_{yy} = \frac{\partial^2 w}{\partial y^2}, \quad \kappa_{xy} = \frac{\partial^2 w}{\partial x \partial y}.$$

Are the curvatures of the deflected surface

### Constitutive Equations

From Hooke's law,

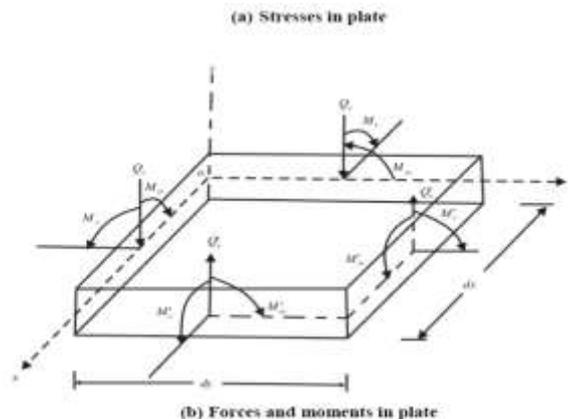
$$\sigma = [D] \varepsilon$$

Where,

$$[D] = \frac{E}{(1-\nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$

Here,  $[D]$  is equal to the value defined for 2D solids in plane stress condition (i.e.,  $\sigma_z = 0$ )

### Calculation of moments and shear forces



Considering a plate element of  $dx \times dy$  and with thickness  $t$ , the plate is subjected to external uniformly distributed load  $p$ . For a thin plate, body force of the plate can be converted to an equivalent load and therefore, consideration of separate body force is not necessary.  $\sigma = -z [D] \Delta w$  It is observed from the above relation that the normal stresses are varying linearly along thickness of the plate. Hence the moments on the cross section can be calculated by integration.

$$M = \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \int_{-t/2}^{t/2} \sigma z dt = - \left( \int_{-t/2}^{t/2} z^2 dt \right) [D] \Delta w = - \frac{t^3}{12} [D] \Delta w$$

On expansion, we have

$$M_x = - \frac{Et^3}{12(1-\nu^2)} \left( \frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} \right) = D_p (\chi_x + \nu \chi_y)$$

$$M_y = - \frac{Et^3}{12(1-\nu^2)} \left( \frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2} \right) = D_p (\chi_y + \nu \chi_x)$$

$$M_{xy} = M_{yx} = \frac{Et^3}{12(1+\nu)} \frac{\partial^2 w}{\partial x \partial y} = - \frac{D_p(1-\nu)}{2} \chi_{xy}$$

Where  $D_p$  is known as flexural rigidity of the plate is given by

$$D_p = \frac{Et^3}{12(1-\nu^2)}$$

Let consider the bending moments vary along the length and breadth of the plate as a function of  $x$  and  $y$ . Thus if  $M_{xacts}$  on one side of the element  $M_x = M_x + \frac{\partial M_x}{\partial x} dx$  acts on the opposite side; Considering equilibrium of the plate element, the equations for forces can be obtained as

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + p = 0$$

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} = Q_x$$

$$\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} = Q_y$$

Substituting the equations for moments, we have

$$Q_x = -D_p \frac{\partial}{\partial x} \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)$$

$$Q_y = -D_p \frac{\partial}{\partial y} \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)$$

Substituting the force equations into the moments equations, we have

$$\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = -\frac{p}{D_p}$$

**Governing Equation for Deflection of Plates**

$$\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \nabla^2 w = \frac{q}{D}$$

$$\nabla^2(\nabla^2 w) = \frac{q}{D}$$

$$\nabla^4 w = \frac{q}{D}$$

$$\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D}$$

(Ramadas Chennamsetti)

**III. MATERIAL**

**The Type of aircraft to be considered**

Airbus A320 (2011) single aisle passenger aircraft is one of the most modern aircraft in Airbus industry. It is a commercial aircraft with high passenger capacity, and it is mostly used for domestic routes. Airbus is one of the leading aircraft manufacturers in the world with the most modern and comprehensive product line with unbeatable fuel efficiency. It has a swept wing according to shape, middle wing according to the position of the wing on the aircraft and bi – wing according to the number of wings.

**Geometry Of The Aircraft Wing**

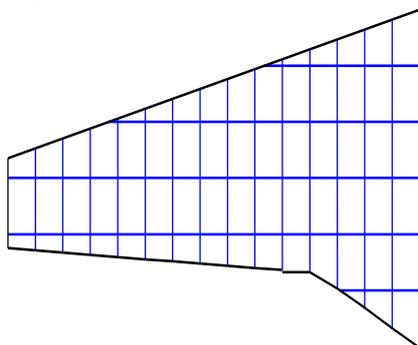
Geometric Scaling of Airbus A 320 Wing:

**TABLE 2: SCALED DIMENSION OF WING (1:5)**

Semi Span	3000mm
Root Chord	1180mm
Tip Chord	192sq.mtrs
Wing Area	763mm
Mac (Location From Root)	250
Sweep Angle L.E	0°
Sweep Angle T.E	170°
Twist Root	3.7°
Twist Tip	0.8°

(PungotiSharadaVani et al., 2014)

The original scaled dimensions (1:5) of the A320 wing are shown in Table 2 above. We know the values of root and tip chord lengths, sweep angles at the leading and trailing edges. The A 320 wing AutoCAD drawing is shown in Fig.3.



**Fig.3** Final Meshed Model of Wing

By using AutoCAD software, the wing is designed according to required specifications. We have divided the wing into 15 stations at a distance of 200 mm. The very purpose of the division of wing is to give varying thickness at each division.

**TABLE 3: CHORD AND THICKNESS AT EACH STATION (1:5)**

S/No	Chord (mm)	Thickness (mm)
0	1186.6	141.98
1	1083.029	130.17
2	979.42	118.61
3	875.3	105.24
4	783	94.11
5	740.56	89.69
6	698.1	83.91
7	656	78.85
8	613.5	73.76
9	571	68.6
10	529.1	63.58
11	486.2	58.44
12	444	53.36
13	401.5	48.26
14	359	43.211
15	192	23.07

(PungotiSharadaVani et al., 2014)

#### IV. FINITE ELEMENT ANALYSIS

##### Programming

The application of computer by the finite element method analysis of stresses for plate bending is written in a programming language known as JAVA. The first part is the main program that carries out the computation of stress analysis on the aircraft wing which is taken as a plate. The second part is the graphics user interface (GUI) will be created that will make the program a computer interactive model which drives the finite element method program. This part forms the interface that generates data input. The computer interactive model has the following steps for problem solving:

**Modelling:** Includes the system geometry definition and material property selection. In this step user can draw either 2D representation of the problem.

**Meshing:** This step involves discretizing the model according to predefined geometric element.

**Solution:** This step involves applying boundary conditions and loads to the system and solves the problem.

**Post processing:** This involves plotting nodal solutions (unknown parameters), which may be of displacements/stresses/reactive forces etc.

V. RESULTS AND DISCUSSION

Sample Runs

Case One

The properties inputted include Young’s Modulus  $E = 72 \times 10^9$  Pa, Poisson ratio  $\nu = 0.3$ , thickness  $t = 0.5m$ , load = 10KPa (Uniform Loading along Y) and geometry.

Table 4 Case One: Geometry and Boundary Condition

SIDE NO	DIMENSION	BOUNDARY
1	5900mm	Fixed
3	1606.9072mm	Free
4	4060.7881mm	Free, Loaded
5	4219.7208mm	Free
6	11149.6092mm	Free, Loaded
7	11008.3554mm	Free

Results

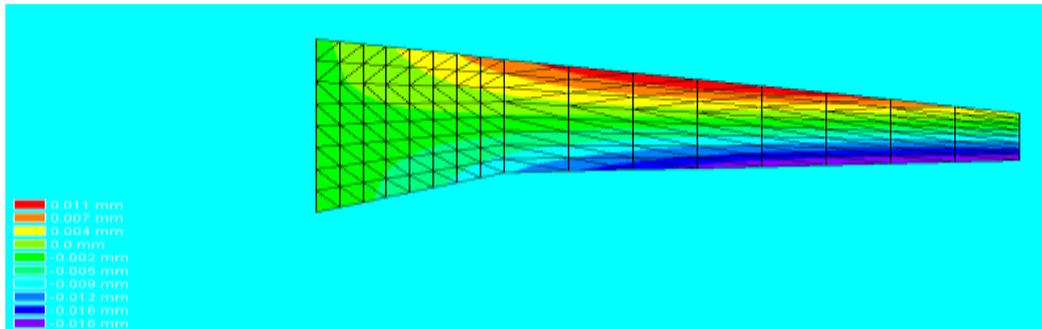


Fig 4: Case One- Displacement Along X – Axis

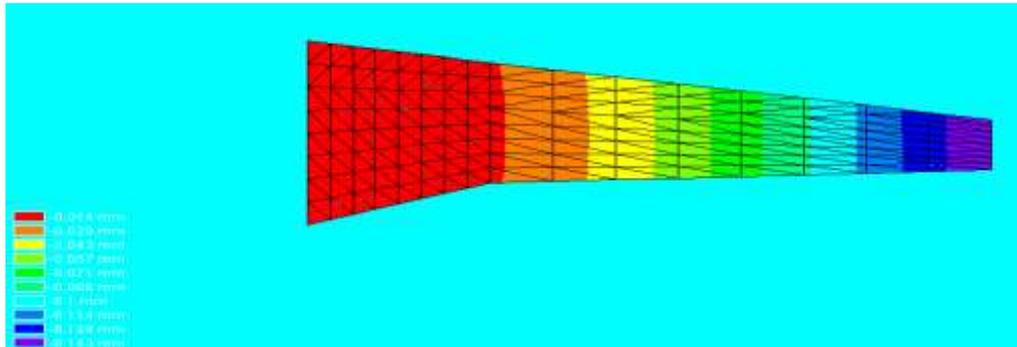


Fig 5: Case One- Displacement Along Y –Axis

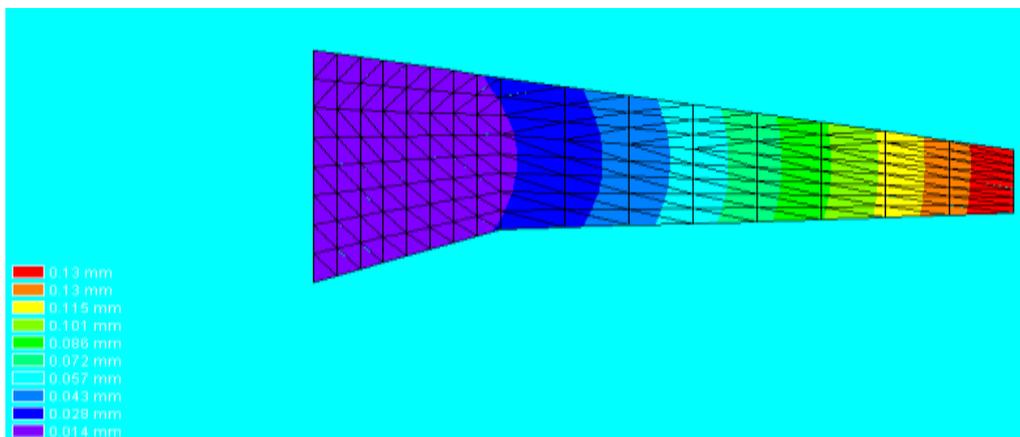


Fig 6: Case One- Displacement Along XY – Axis

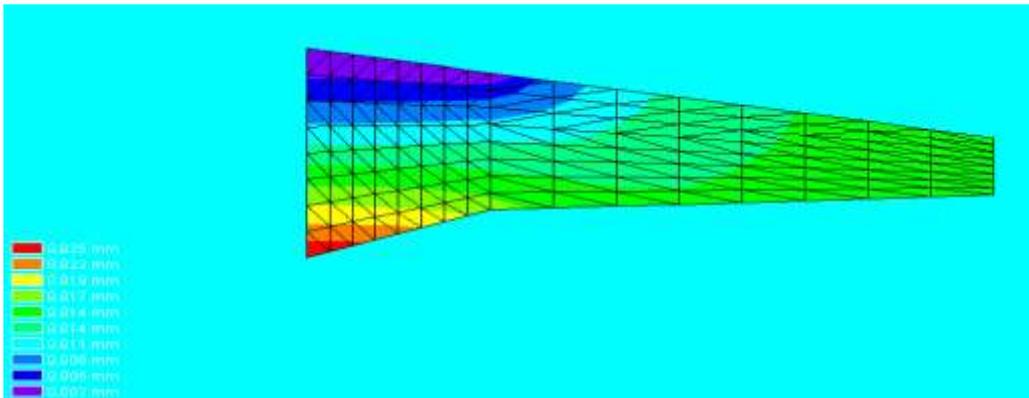
**Case Two**

The properties inputed include Young’s Modulus  $E = 72 \times 10^9$  Pa, Poisson ratio  $\nu = 0.3$ , thickness  $t = 0.5$ m, load = 15KPa (Uniform Loading along X) and geometry

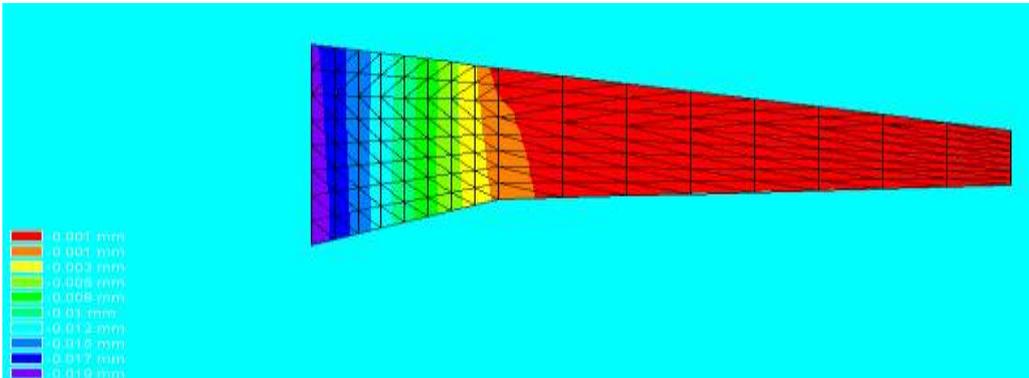
**Table 5** Case Two: Loading and Boundary Condition

SIDE NO	LOADING	BOUNDARY
1	0	Free
3	0	Free
4	0	Roller Support
5	15KPa	Free
6	0	Roller Support
7	15KPa	Free

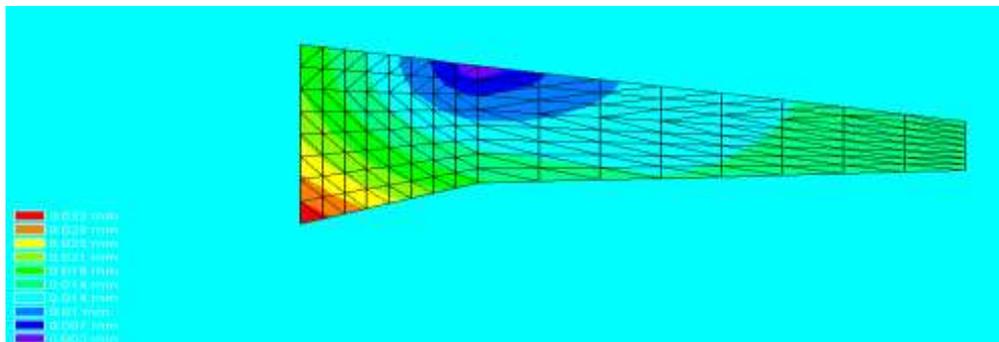
**Results**



**Fig 7:** Case Two-Displacement Along X – Axis



**Fig 8:** Case Two-Displacement Along Y – Axis



**Fig 9:** Case Two-Displacement Along XY – Axis

## VI. CONCLUSION

This project work considered analysis of deflection of an aircraft wing under different loading conditions using finite element method. JAVA code was used to develop an interactive computer model which analyses the deflection of an aircraft wing under various loading (uniformly distributed loading and point concentrated loading) conditions at the edges and the nodes and support conditions (fixed support, roller support and free end). Two different cases were analysed and deflection (lateral displacement) along X – axis, Y – Axis and XY – Axis for each cases were presented. It was discovered that the greater the load, the higher the deflection obtained and for edges and nodes with fixed and roller support, higher loadings has to be applied for deflection to occur.

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Odunfa, K. M" Numerical Modeling of Aircraft Wing Deflection under Different Loading Conditions" International Journal of Modern Engineering Research (IJMER), vol. 08, no. 11, 2018, pp 71-79