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On Intuitionistic Fuzzy Πgb-D Sets And Some Of Its Applications

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ABSTRACT: *In this paper we introduce and study new classes of separation axioms with compactness and connectedness by using intuitionistic fuzzy πgb-sets and intuitionistic fuzzy πgb-D-sets.*
KEY WARDS: *IFπgb-D-sets, IFπgb-Di-spaces for i = 0,1,2, IFπgb-D-compact and IFπgb-D-connected.*

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I. INTRODUCTION

The concept of fuzzy set was introduced by L. A. Zadah [13]. The fuzzy concept has invaded almost all branches of mathematics. The concept of fuzzy topological space was introduced and develped by C. L. Chang [4]. Atanasov [3] was introduced The concept of intuitionistic fuzzy set, as a generalization of fuzzy set. This approach provided a wide field to the generalization of various concepts of fuzzy mathematics. In 1997 Coker[7] defined intuitionistic fuzzy topological spaces. Recently many concepts of fuzzy topological space have been extended in intuitionistic fuzzy (IF) topological spaces. D. Sreeja and C. Janaki [11] intrduced the concept of πgb-D-sets and Some Low Separation Axioms. Amal M. Al-Dowais and AbdulGawad A. Al-Qubati [2] studied the concept of slightly πgb-continuous functions in intuitionistic fuzzy topological spaces. In this paper we introduce and study the concepts of intuitionistic fuzzy πgb-D-sets. We also introduced intuitionistic fuzzy πgb-D_i-spaces for i = 0,1,2, compactness, connectedness and discussed their properties.

II. PRELIMINARIES

Definition 2.1 [3] Let X be a non empty fixed set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form $A = \{< x,\mu_A(x),\nu_A(x) > : x \in X\}$, where the function $\mu_A(x) : X \to [0,1]$ and $\nu_A(x) : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \le \mu_A(x) + \nu_A(x) \le 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

Definition 2.2 [3] Let A and B be IFSs of the form $A = \{< x, \mu_A(x), \nu_A(x) > : x \in X\}$ and $B = \{< x, \mu_B(x), \nu_B(x) >: x \in X\}$. Then :

(a) $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ for all $x \in X$.
(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.
(c) $A^c = \{< x, \nu_A(x), \mu_A(x) > : x \in X\}$.
(d) $A \cap B = \{< x, \mu_A(x) \wedge \mu_B(x),\nu_A(x) \vee \nu_B(x) > : x \in X\}$.
(e) $A \cup B = \{< x, \mu_A(x) \vee \mu_B(x),\nu_A(x) \wedge \nu_B(x) >: x \in X\}$.
(f) $A-B = \{< x, \min(\mu_A(x), \nu_B(x)), \max(\nu_A(x), \mu_B(x)) > : x \in X\}$.
(g) $0_\sim = \{< x, 0, 1 >: x \in X\}$ and $1_\sim = \{< x, 1, 0 >: x \in X\}$.
(h) $0^c{}_\sim = 1_\sim$ and $1^c{}_\sim = 0_\sim$.

Definition 2.3 [5] Let α , $\beta \in [0,1]$ such that $\alpha+\beta \le 1$. An intuitionistic fuzzy point (IFP) $p_{(\alpha,\beta)}$ is intuitionistic fuzzy set defined by

$$p_{(\alpha,\beta)} = \begin{cases} (\alpha,\beta) & if \quad x = p \\ (0,1) & if \quad otherwise \end{cases}$$

In this case, p is called the support of $p_{(\alpha,\beta)}$ and α , β are called the value and no value of $p_{(\alpha,\beta)}$ respectively.

Clearly an intuitionistic fuzzy point can be represented by an ordered pair of fuzzy point as follows: $p_{(\alpha,\beta)} = (p_\alpha , p_{(1-\beta)})$

In IFPp(α,β) is said to belong to an IFS A = {< x,$\mu_A(x)$,$\nu_A(x)$ >: x ∈ X} denoted by $p_{(\alpha,\beta)} \in A$, if $\alpha \le \mu_A(x)$ and $\beta \ge \nu_A(x)$.

Definition 2.4 [3] Let X and Y be two nonempty sets and f : X → Y be a function. Then:

(a) If B = {<y, $\mu_B(y)$, $\nu_B(y)$ >: y ∈ Y} is an IFS in Y , then the preimage of B under f denoted by $f^{-1}(B)$ is the IFS in X defined by $f^{-1}(B)$ = {< x , $f^{-1}(\mu_B)(x)$, $f^{-1}(\nu_B)(x)$ >: x ∈ X}.

(b) If A = {< x, $\mu_A(x)$, $\nu_A(x)$ > : x ∈ X} is an IFS in X, then the image of A under f denoted by f(A) is the IFS in Y defined by f(A) = {< y , f (μ_A)(y) , 1− f (1−ν_A)(y) >: y ∈ Y} where,.

$$f(\mu_A)(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \mu_A(x) & if \quad f^{-1}(y) \neq 0_\sim \\ 0 & if \quad otherwise \end{cases}$$

$$1 - f(1-\nu_A)(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \nu_A(x) & if \quad f^{-1}(y) \neq 0_\sim \\ 1 & if \quad otherwise \end{cases}$$

Definition 2.5 [6] An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms :

(i) $0_\sim , 1_\sim \in \tau$

(ii) $G_1 \cap G_2$ for any G_1 , G_2 in τ .

(iii) $\bigcup G_i \in \tau$ for any family $\{G_i : i \in J\} \subseteq \tau$

In this case the pair (X,τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFT in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X. the complement A^c of an IFOS A in an IFTS (X,τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.6 [7] A subset A of an intuitionistic fuzzy space X is said to be clopen if it is intuitionistic fuzzy open set and intuitionistic fuzzy closed set.

Definition 2.7 [4] Let (X,τ) be an IFTS and A = {< x,$\mu_A(x)$,$\nu_A(x)$ >: x ∈ X} be an IFS in X. Then the intuitionistic fuzzy interior and an intuitionistic fuzzy clousre are defined by :

$$int(A) = \bigcup \{G : G \text{ is an IFOS in X and } G \subseteq A\}$$
$$cl(A) = \bigcap \{K : K \text{ is an IFCS in X and } A \subseteq K\}$$

Definition 2.8 An IFS A of an IFTS (X,τ) is an :

1. Intuitionistic fuzzy regular open set (IFROS in short) if int(cl(A)) = A. [7]
2. Intuitionistic fuzzy regular closed set (IFRCS in short) if cl(intl(A)) = A. [7]
3. Intuitionistic fuzzy π-open set (IFπOS in short) if the finite union of intuitionistic fuzzy regular open sets. [10]
4. Intuitionistic fuzzy π-closed set (IFπCS in short) if the finite intersection of intuitionistic fuzzy regular closed sets. [10]

5. Intuitionistic fuzzy generalized open set (IFGOS in short) if F ⊆ int(A) whenever F ⊆ A and F is an IFCS in X. [12]
6. Intuitionistic fuzzy generalized closed set (IFGCS in short) if cl(A) ⊆ U whenever A ⊆ U and U is an IFOS in X . [12]
7. Intuitionistic fuzzy b-open set (IFbOS in short) if A ⊆ cl(int(A)) ∪ int(cl(A)). [1]
8. Intuitionistic fuzzy b-closed set (IFbCS in short) if cl(int(A)) ∪ int(cl(A)) ⊆ A. [1]
9. Intuitionistic fuzzy semi-open set (IFSOS in short) if A ⊆ cl(int(A)). [7]
10. Intuitionistic fuzzy semi-closed set (IFSCS in short) if int(cl(A)) ⊆ A. [7]
11. Intuitionistic fuzzy α-open set (IFαOS in short) if A ⊆ int(cl(int(A))). [7]
12. Intuitionistic fuzzy α-closed set (IFαCS in short) if cl(int(cl(A))) ⊆ A. [7]
13. Intuitionistic fuzzy pre-open set (IFPOS in short) if A ⊆ int(cl(A)). [7]
14. Intuitionistic fuzzy pre-closed set (IFPCS in short) if cl(int(A)) ⊆ A. [7]

Definition 2.9 [9] Let (X,τ) be an IFTS and A be an IFS in X. Then the intuitionistic fuzzy b-interior and an intuitionistic fuzzy b-clousre are defined by :

$$\mathrm{bint}(A) = \bigcup \{G : G \text{ is an IFbOS in X and } G \subseteq A\}$$
$$\mathrm{bcl}(A) = \bigcap \{K : K \text{ is an IFbCS in X and } A \subseteq K\}$$

Theorem 2.10 Let A be an intuitionistic fuzzy set of an IFTS (X,τ), then :
(i) $\mathrm{bcl}(A) = A \bigcup [\mathrm{int}(\mathrm{cl}(A)) \bigcap \mathrm{cl}(\mathrm{int}(A))]$
(ii) $\mathrm{bint}(A) = A \bigcap [\mathrm{int}(\mathrm{cl}(A)) \bigcup \mathrm{cl}(\mathrm{int}(A))]$

Definition 2.11 [2] An IFS A of an IFTS (X,τ) is an :
1. Intuitionistic fuzzy πgb-open set (IFπgbOS in short) if F ⊆ bint(A) whenever F ⊆ A and F is an IFπCS in X.
2. Intuitionistic fuzzy πgb-closed set (IFπgbCS in short) if bcl(A) ⊆ U whenever A ⊆ U and U is an IFπOS in X.

Definition 2.12 [2] Let f be a function from an IFTS (X,τ) into an IFTS (Y,σ). Then f is said to be an intuitionistic fuzzy πgb-continuous if $f^{-1}(F)$ is an intuitionistic fuzzy πgb-closed in (X,τ) for every intuitionistic fuzzy closed set F of (Y,σ).

Definition 2.13 [2] Let f be a function from an IFTS (X,τ) into an IFTS (Y,σ). Then f is said to be an intuitionistic fuzzy πgb-irresolute if $f^{-1}(F)$ is an intuitionistic fuzzy πgb-closed in X for every intuitionistic fuzzy πgb-closed set F of Y .

Definition 2.14 [2] An IFTS (X,τ) is called $(\pi gb - T_0)$ if and only if for each pair of distinct intuitionistic fuzzy points $x_{(\alpha,\beta)}$, $y_{(\nu,\delta)}$ in X there exists an intuitionistic fuzzy πgb-open set U,∈ X such that $x_{(\alpha,\beta)} \in U, y_{(\nu,\delta)} \notin U$.

Definition 2.15 [2] An IFTS (X,τ) is called $(\pi gb - T_1)$ if and only if for each pair of distinct intuitionistic fuzzy points $x_{(\alpha,\beta)}$, $y_{(\nu,\delta)}$ in X there exists intuitionistic fuzzy πgb-open sets U,V ∈ X such that $x_{(\alpha,\beta)} \in U, y_{(\nu,\delta)} \notin U$ and $y_{(\nu,\delta)} \in V, x_{(\alpha,\beta)} \notin V$.

Definition 2.16 [2] An IFTS (X,τ) is said to be $\pi gb{-}T_2$ or πgb-Hausdorff if for all pair of distinct intuitionistic fuzzy points x(α,β) , y(ν,δ) in X there exits IFπgb open sets U,V ∈ X such that $x_{(\alpha,\beta)} \in U, y_{(\nu,\delta)} \in V$ and $U \cap V = 0_\sim$.

Main Results And Applications

Definition 3.1 An IFS A of an IFTS (X,τ) is an :
i) Intuitionistic fuzzy D-set if there exits IF open sets U,V in (X,τ) such that $U \neq 1_\sim$ and $A = U - V$.
ii) Intuitionistic fuzzy semi-D-set if there exits IF semi-open sets U,V in (X,τ) such that $U \neq 1_\sim$ and $A = U - V$.
iii) Intuitionistic fuzzy α-D-set if there exits IF α-open sets U,V in (X,τ) such that $U \neq 1_\sim$ and $A = U - V$.
iv) Intuitionistic fuzzy pre-D-set if there exits IF pre-open sets U,V in (X,τ) such that $U \neq 1_\sim$ and $A = U - V$.
v) Intuitionistic fuzzy b-D-set if there exits IF b-open sets U,V in (X,τ) such that $U \neq 1_\sim$ and $A = U - V$.

Remark 3.2 Clearly every IFOS (respectively IFSO, IFαOS, IFPOS, IFbOS) U different from $1_\sim$ is an IFD-set

(respectively IFSD-set, IFαDset, IFPD-set, IFb-set) if A = U and V = $0_\sim$.

The converse of the above remark is not true.

Example 3.3 Let X ={a,b}, A = {< a, 0.1, 0.9 >,< b, 0.8, 0.2 >}, B = {< a, 0.2, 0.8 >,< b, 0.9, 0.1 >}, and IFTS τ = {$0_\sim$,$1_\sim$, A, B}. Then, D = {< a, 0.1, 0.9 >,< b, 0.1, 0.9 >} is an (IF-D-set, IFα-D-set, IFS-D-set, IFP-Dset and IFb-D-set) but not an (IFOS, IFαOS, IFSOS, IFPOP and IFbOS).

Since U = {< a, 0.1, 0.9 >,< b, 0.8, 0.2 >} ≠ $1_\sim$ and V = {< a, 0.2, 0.8 > ,< b, 0.9, 0.1 >} are (IFOSs, IFαOSs, IFSOSs, IFPOSs and IFbOSs) in (X,τ), $U - V = U \cap V^c = D$.

Theorem 3.4 (i) Every IFD-set is an IFαD-set , IFSD-set , IFPD-set and IFbD-set .
(ii) Every IFαD-set is an IFSD-set , IFPD-set and IFbD-set .
(iii) Every IFSD-set and IFPD-set is an IFbD-set.

Proof. Obvious. ■

Definition 3.5 An IFS A of an IFTS (X,τ) is called IF πgb-D-set if there exits IFπgb open sets U,V in (X,τ) such that U ≠ $1_\sim$ and A = U −V .

Remark 3.6 Every IF πgb-open set U different from $1_\sim$ is an IFπgb-D-set if A = U and V = $0_\sim$.

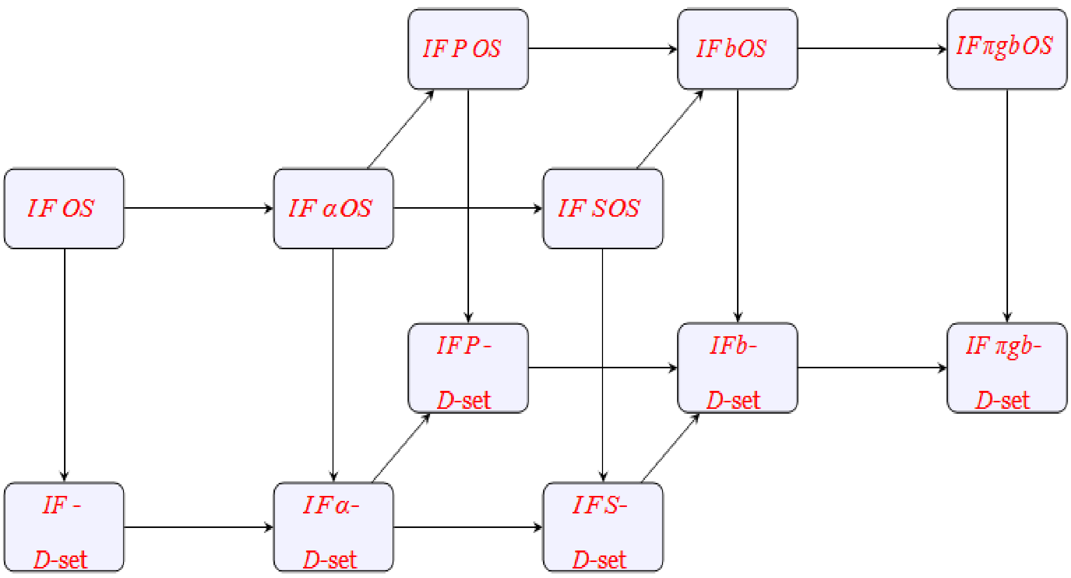
Example 3.7 Let X ={a,b,c}, and A = {< a, 1, 0 >,< b, 0, 1 >,< c, 0, 1 > }, B = {< a, 0, 1 >,< b, 0.8, 0.2 >,< c, 0.3, 0.7 >}, C = {< a, 1, 0 >,< b, 0.8, 0.2 >,< c, 0.3, 0.7 >}, and is IFTS τ = {0∼,1∼,A,B,C}. Then D = {< a, 0, 1 >,< b, 0.2, 0.8 >,< c, 0.7, 0.3 >} is an IFπgb-D-set but not IF πgb-open set. Since U = {< a,1,0 >,< b,0.4,0.6 >,< c,0.7,0.3 >} ≠ $1_\sim$ and V = {< a, 1, 0 >,< b, 0.8, 0.2 >,< c, 0.1, 0.9 >} are IF πgb-open sets in (X,τ), $U - V = U \cap V^c = D$, then D is an IFπgb-D-set but not IF πgb-open set. Since C^c (π-closed set)⊆ D and $C^c \subseteq$ bint(D) = $0_\sim$.

Theorem 3.8 Every IFD-set, IFαD-set, IFPD-set, IFSD-set, IFbD-set is an IFπgb-D-set.

Proof. Since every IFOS, IFαOS, IFPOS, IFSOS and IFbOS is an IFπgbOS. Then the proof is dirctly clear. ■

The converse of the above theorem is not true.

Example 3.9 Let X ={a,b,c}, and A = {< a, 1, 0 >,< b,0,1 >,< c,0,1 >}, B = {< a,1,0 >,< b, 1, 0 >,< c, 0, 1 >}, then τ = {$0_\sim$,$1_\sim$, A , B}. Then D = {< a, 0, 1 >,< b, 1, 0 >,< c, 1, 0 >} is an IFπgb-D-set but not IFD-set, IFαD-set and IFSD-set.



Definition 3.10 X is said to be an :

1. IFπgb- D_0 if and only if for each pair of distinct intuitionistic fuzzy points, $x_{(\alpha,\beta)}$, $y_{(v,\delta)}$ in X there exists an intuitionistic fuzzy πgb-D-set U∈ X such that $x_{(\alpha,\beta)} \in U$, $y_{(v,\delta)} \notin U$.
2. IFπgb-D_1 if and only if for each pair of distinct intuitionistic fuzzy points $x_{(\alpha,\beta)}$,$y_{(v,\delta)}$ in X there exists intuitionistic fuzzy πgb-D-sets U,V ∈ X such that $x_{(\alpha,\beta)} \in U, y_{(v,\delta)} \notin U$ and, $y_{(v,\delta)} \in V, x_{(\alpha,\beta)} \notin V$.
3. IFπgb-D_2(IFπgb-D-Hausdorf) if for all pair of distinct intuitionistic fuzzy points $x_{(\alpha,\beta)}$, $y_{(v,\delta)}$ in X there exits IFπgb-D-sets U,V ∈ X such that $x_{(\alpha,\beta)} \in U, y_{(v,\delta)} \in V$ and $U \cap V = 0_\sim$.

Theorem 3.11 **1.** If (X,τ) is IF πgb-T_i , then (X,τ) is IF πgb-D_i ; i = 0,1,2.
2. If (X,τ) is IF πgb-D_i , then (X,τ) is IF πgb-D_{i-1} ; i = 1,2. 3. If (X,τ) is IF πgb-T_i, then (X,τ) is IF πgb-T_{i-1} ; i = 1,2.

Theorem 3.12 For an intuitionistic fuzzy topological space (X,τ). the following statements hold:
1. (X,τ) is IF πgb-D_0 if and only if it is IF πgb-T_0.
2. (X,τ) is IF πgb-D_1 if and only if it is IF πgb-D_2.

Proof. (i) ⇒ Let (X,τ) be IF πgb-D_0. Then for any two distinct IF points $x_{(\alpha,\beta)}$,$y_{(v,\delta)}$ in X, let $x_{(\alpha,\beta)}$ belongs to IF πgbD-set G where $y(v,\delta) \notin G$. Let $G = U_1 - U_2$ where $U_1 \neq 1_\sim$ and U_1,U_2 are IF πgbOSs in X. Then $x_{(\alpha,\beta)} \in U_1$ for $y_{(v,\delta)} \notin G$ we have two cases : (a) $y_{(v,\delta)} \notin U_1$ (b) $y_{(v,\delta)} \in U_2$ and $y_{(v,\delta)} \in U_2$. In case (a), $x_{(\alpha,\beta)} \in U_1$ but $y_{(v,\delta)} \notin U_1$; in case (b) $y_{(v,\delta)} \in U_2$ and $x_{(\alpha,\beta)} \notin U_2$. Hence (X,τ) is IF πgb-$T_0$.

⇐ By theorem (3.11) (1)
(ii) ⇒ Suppose (X,τ) is an IF πgb-D_1, then for each distinct IFP's pair $x_{(\alpha,\beta)}$,$y_{(v,\delta)}$ in (X,τ), we have IF πgbD-sets G_1 and G_2 such that $x_{(\alpha,\beta)} \in G_1$ and $y_{(v,\delta)} \notin G_1$, $x_{(\alpha,\beta)} \notin G_2$ and $y_{(v,\delta)} \in G_2$. Let $G_1 = U_1 - U_2$ and $G_2 = U_3 - U_4$. By $x_{(\alpha,\beta)} \notin G_2$, it follows that either $x_{(\alpha,\beta)} \notin U_3$ or $x_{(\alpha,\beta)} \in U_3$ and $x_{(\alpha,\beta)} \in U_4$.

Now we have two cases : (i) $x_{(\alpha,\beta)} \notin U_3$. By $y_{(v,\delta)} \notin G_1$ we have two subcases : (a) $y_{(v,\delta)} \notin U_1$; By $x_{(\alpha,\beta)} \in U_1 - U_2$, it follows that $x_{(\alpha,\beta)} \in U_1 - (U_2 \bigcup U_3)$ and by $y_{(v,\delta)} \in U_3 - U_4$, we have $y_{(v,\delta)} \in U_3 - (U_1 \bigcup U_4)$. Hence $(U_1 - (U_3 \bigcup U_2)) \bigcap (U_3 - (U_1 \bigcup U_4)) = 0_\sim$. (b) $y_{(v,\delta)} \in U_1$ and $y_{(v,\delta)} \in U_2$, we have $x_{(\alpha,\beta)} \in U_1 - U_2$,$y_{(v,\delta)} \in U_2 \Rightarrow (U_1 - U_2) \bigcap U_2 = 0_\sim$. (ii) $x_{(\alpha,\beta)} \in U_3$ and $x_{(\alpha,\beta)} \in U_4$. We have $y(v,\delta) \in U_3 - U_4$, $x_{(\alpha,\beta)} \in U_4 \Rightarrow (U_3 - U_4) \bigcap U_4 = 0_\sim$. Thus (X,τ) is IF πgb-$D_2$. ⇐ By theorem (3.11) (2) ■

Theorem 3.13 If (X,τ) is IF πgb-D_1, then it is IF πgb-T_0.

Proof. By theorem (3.11) and theorem (3.12). ■

$$
\begin{array}{ccccc}
IF\ \pi gb\text{-}T_2 & \rightarrow & IF\ \pi gb\text{-}T_1 & \rightarrow & IF\ \pi gb\text{-}T_0 \\
\downarrow & & \downarrow & & \updownarrow \\
IF\ \pi gb\text{-}D_2 & \leftrightarrow & IF\ \pi gb\text{-}D_1 & \rightarrow & IF\ \pi gb\text{-}D_0
\end{array}
$$

Theorem 3.14 Let f : (X,τ) → (Y,σ) be an IFπgb-continuous surjective function and S is an IFD-set of (Y,σ), then the inverse image of G is an IFπgb-D-set of (X,τ).

Proof. Let U_1 and U_2 be two IF open sets of (Y,σ). Let $G = U_1 - U_2$ be an IFD-set and $U_1 \neq 1_\sim$. We have $f^{-1}(U_1)$, $f^{-1}(U_2)$ ∈ IFπgbOSs in (X,τ) and $f^{-1}(U_1) \neq 1_\sim$. Hence $f^{-1}(G) = f^{-1}(U_1 - U_2) = f^{-1}(U_1) - f^{-1}(U_2)$. Hence $f^{-1}(G)$ is an IFπgb-D-set. ■

Theorem 3.15 Let f : (X,τ) → (Y,σ) be an IFπgb-irresolute surjection function and E is an IFπgb-D-set of (Y,σ), then the inverse image of E is an IFπgb-D-set of (X,τ).

Proof. Let E be an IFπgb-D-set in (Y,σ). Then there are an IFπgb-open sets U_1 and U_2 in Y such that E = $U_1 - U_2$ and $U_1 \neq 1_{\sim Y}$. Since f is IFπgb irresolute, $f^{-1}(U_1)$ and $f^{-1}(U_2)$ IFπgb-open sets in X. Since $U_1 \neq 1_{\sim Y}$, we have $f^{-1}(U_1) \neq 1_{\sim X}$. Hence $f^{-1}(E) = f^{-1}(U_1 - U_2) = f^{-1}(U_1) - f^{-1}(U_2)$ is an IFπgb-D-set in (X,τ). ■

Theorem 3.16 Let f : (X,τ) → (Y,σ) be an IFπgb-continuous bijective function and (Y,σ) is an IFD_1-space, then (X,τ) is an IFπgb-D_1-space.

Proof. Suppose (Y,σ) is an IFD_1-space. Let $x_{(\alpha,\beta)}$, $y_{(\nu,\delta)}$ be any distinct IFPs in X. Sence f is injective and (Y,σ) is an IFD_1-space, then there exists IFD-sets S_1 and S_2 of (Y,σ) containing $f(x_{(\alpha,\beta)})$ and $f(y_{(\nu,\delta)})$ respectively that $f(x_{(\alpha,\beta)}) \notin S_2$ and $f(y_{(\nu,\delta)}) \notin S_1$. By theorem(3.14) $f^{-1}(S_1)$ and $f^{-1}(S_2)$ are IFπgb-D-sets in X containing $x_{(\alpha,\beta)}$ and $y_{(\nu,\delta)}$ respectively such that $x_{(\alpha,\beta)} \notin f^{-1}(S_2)$ and $y_{(\nu,\delta)} \notin f^{-1}(S_1)$. Hence (X,τ) is an IFπgb-D_1-space. ■

Theorem 3.17 Let (Y,σ) be an IFπgb-D_1 and f : (X,τ) → (Y,σ) is IFπgb irresolute bijective function, then (X,τ) is an IFπgb-D_1-space.

Proof. Suppose (Y,σ) is IFπgb-D_1-space and f is is IFπgb-irresolute bijective function. Let $x_{(\alpha,\beta)}$, $y_{(\nu,\delta)}$ be any distinct IFPs in (X,τ). Sence f is injective and Y is an IFD_1-space, then there exists IFD-sets S_1 and S_2 of Y containing $f(x_{(\alpha,\beta)})$ and $f(y_{(\nu,\delta)})$ respectively that $f(x_{(\alpha,\beta)}) \notin S_2$ and $f(y_{(\nu,\delta)}) \notin S_1$. By theorem(3.15) $f^{-1}(S_1)$ and $f^{-1}(S_2)$ are IFπgb-D-sets in X containing $x_{(\alpha,\beta)}$ and $y_{(\nu,\delta)}$ respectively such that $x_{(\alpha,\beta)} \notin f^{-1}(S_2)$ and $y_{(\nu,\delta)} \notin f^{-1}(S_1)$. Hence (X,τ) is an IFπgb-D_1-space. ■

Theorem 3.18 A topological space (X,τ) is IFπgb-D_1-space if for each pair of distinct IFPs $x_{(\alpha,\beta)}$, $y_{(\nu,\delta)}$ in (X,τ), there exists an IFπgb-continuous surjective function f : (X,τ) → (Y,σ) where (Y,σ) is an IFD_1 space such that $f(x_{(\alpha,\beta)})$ and $f(y_{(\nu,\delta)})$ are distinct.

Proof. Let $x_{(\alpha,\beta)}$, $y_{(\nu,\delta)}$ be any distinct IFPs in (X,τ) and f is an IFπgb-continuous surjection function of an IFS (X,τ) onto an IFD_1 space (Y,σ) such that $f(x_{(\alpha,\beta)}) \neq f(y_{(\nu,\delta)})$. Hence there exists disjoint IFD-sets U_1 and U_2 in (Y,σ) such that $f(x_{(\alpha,\beta)}) \in U_1$ and $f(y_{(\nu,\delta)}) \in U_2$. Since f is an IFπgb-continuous surjective function, by theorem (3.14) $f^{-1}(U_1)$ and $f^{-1}(U_2)$ are disjoint IFπgb-D-sets in X containing $x_{(\alpha,\beta)}$ and $y_{(\nu,\delta)}$ respectively. Hence (X,τ) is an IFπgbD_1-space. ■

Theorem 3.19 (X,τ) is IFπgb-D_1-space if and only if for each pair of distinct IFPs $x_{(\alpha,\beta)}$, $y_{(\nu,\delta)} \in X$, there exists an IFπgb-irresolute surjective function f : (X,τ) → (Y,σ), where (Y,σ) is an IFπgb-D_1 space such that $f(x_{(\alpha,\beta)})$ and $f(y_{(\nu,\delta)})$ are distinct.

Proof. Necessity. For every pair of distinct IFP's of (X,τ), it suffices to take the identity function on (X,τ).
Sufficiency. Let $x_{(\alpha,\beta)}$, $y_{(\nu,\delta)}$ be any distinct IFPs in (X,τ) there exists an IFπgb-irresolute surjection function of an IFS (X,τ) onto an IFπgb-D_1 space (Y,σ) such that $f(x_{(\alpha,\beta)}) \neq f(y_{(\nu,\delta)})$. Hence there exists disjoint IFπgb-D-sets U_1 and U_2 in (Y,σ) such that $f(x_{(\alpha,\beta)}) \in U_1$ and $f(y_{(\nu,\delta)}) \in U_2$. Since f is an IFπgb-irresolute surjection function, by theorem (3.15) $f^{-1}(U_1)$ and $f^{-1}(U_2)$ are disjoint IFπgb-D-sets in (X,τ) containing $x_{(\alpha,\beta)}$ and $y_{(\nu,\delta)}$ respectively. Hence (X,τ) is an IFπgb-D_1-space. ■

Definition 3.20 An IFTS (X,τ) is said to be IFπgb-D-disconnected if there exists IFπgb-D-open set A , B in (X,τ) such that $A \neq 0_\sim$, $B \neq 0_\sim$, such that $A \cup B = 1_\sim$ and $A \cap B = 0_\sim$. If X is not IFπgb-D-disconnected then it is said to be IFπgb-D-connected.

Theorem 3.21 Let f : (X,τ) → (Y,σ) be an IFπgb-D-continuous surjection, and (X,τ) is an IFπgb-D-connected, then (Y,σ) is an IFD-connected.

Proof. Assume that (Y,σ) is not IFD connected then there exists nonempty IFOS's A and B in (Y,σ) such that $A \bigcup B = 1_\sim$ and $A \cap B = 0_\sim$. Therefore, A and B are intuitionistic fuzzy open sets in Y . Since f is IFπgb-D-continuous $C = f^{-1}(A) \neq 0_\sim$, $D = f^{-1}(B) \neq 0_\sim$, which are IFπgb-D-open sets in X. And $f^{-1}(A) \cup f^{-1}(B) = f^{-1}(A \bigcup B) = f^{-1}(1_\sim) = 1_\sim$ which implies $C \bigcup D = 1_\sim$, $f^{-1}(A) \cap f^{-1}(B) = f^{-1}(A \cap B) = f^{-1}(0_\sim) = 0_\sim$ which implies $C \cap D = 0_\sim$. Thus X is IFπgb-D-disconnected, which is a contradiction to our hypothesis. Hence (Y,σ) is an IFD-connected. ■

Definition 3.22 Let (X,τ) be an IFTS. A family {< x, $\mu_{Gi}(x)$, $\nu_{Gi}(x)$ > : i ∈ J} of an intuitionistic fuzzy πgb-D-sets in (X,τ) satisfies the condition $1_{\sim}$ = S{< x, $\mu_{Gi}(x)$, $\nu_{Gi}(x)$ > ; i ∈ J} is called an intuitionistic fuzzy πgb-D cover of (X,τ). A finite subfamily of an intuitionistic fuzzy πgb-D cover {< x, $\mu_{Gi}(x)$, $\nu_{Gi}(x)$ > : i ∈ J} of (X,τ) which is also an intuitionistic fuzzy πgb-D cover of (X,τ) is called a finite subcover of {< x, $\mu_{Gi}(x)$, $\nu_{Gi}(x)$ >: i ∈ J}.

Definition 3.23 An IFTS (X,τ) is called intuitionistic fuzzy πgb-D-compact if each intuitionistic fuzzy πgb-D-set cover of (X,τ) has a finite subcover of (X,τ).

Theorem 3.24 Let f : (X,τ) → (Y,σ) be an IFπgb-continuous surjection, (X,τ) is an intuitionistic fuzzy πgb-D-compact space, then (Y,σ) is an intuitionistic fuzzy D-compact.

Proof. Let f : (X,τ) → (Y,σ) be an IFπgb-continuous function from an intuitionistic fuzzy πgb-D-compact space (X,τ) onto an intuitionistic fuzzy topological space (Y,σ). Let {A_i ; i ∈ J} be an intuitionistic fuzzy πgb-D cover of Y , then {$f^{-1}(A_i)$; i ∈ J} is an intuitionistic fuzzy πgb-D cover of X. Since X is intuitionistic fuzzy πgb-D-compact it has finite intuitionistic fuzzy subcover say {$f^{-1}(A_1)$, $f^{-1}(A_2)$,............., $f^{-1}(A_n)$}. Since f is onto, {A_1, A_2, ..., A_n} is an intuitionistic fuzzy cover of (Y,σ), by intuitionistic fuzzy D cover has a finite subcover and so (Y,σ) is intuitionistic fuzzy D-compact. ■

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