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On Fuzzy Soft Relations and Similarity of Fuzy Soft Sets

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ABSTRACT: *The purpose of this paper is to study the notion of relations on fuzzy soft sets and similarity between two fuzzy soft sets. We have put forward the notions of fuzzy soft reflexive relations, fuzzy soft symmetric relations, fuzzy soft transitive relations and fuzzy soft equivalence relations. Some properties of composition of fuzzy soft relations have been discussed in our work. An effort has been made to introduce the notion of cardinality of a fuzzy soft set followed by similarity of two fuzzy soft sets. Some properties of similarity of fuzzy soft sets have been put forward followed by a decision making problem.*
KEY WORDS: *Fuzzy Soft Set, Fuzzy Soft Relation, Similarity of Fuzzy Soft Sets.*

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I. INTRODUCTION

Uncertainty plays an important role in everyday life. Zadeh [9] initiated the theory of fuzzy sets to deal with uncertainty. In 1999, Molodtsov [6] developed the concept of soft set and it was a new mathematical tool to deal with uncertainties. The notion of soft set has now gained attention of researchers. Works are going on the development of soft set theory. In 2001, Maji et al. [4] developed the concept of fuzzy soft sets. Fuzzy soft set is a hybrid model which uses both soft sets and fuzzy sets. The results of Maji were later revised and improved by Ahmad and Kharal [1]. Works have been done to formulate the notion of relations on fuzzy soft sets, eg. Borah et.al [2], Chaudhuri et.al [3], Sut [8] etc.

Similarity measure has several applications in the field of pattern recognition, region extraction, image processing etc. Mazumder [5] worked a lot in developing results on similarity of fuzzy soft sets. In our work, an effort has also been done to develop some results on fuzzy soft relations and similarity of fuzzy sets followed by a decision problem.

1.1 Preliminaries

In this section, we first recall the basic definitions related to fuzzy soft sets which would be used in the sequel.

Definition 1.1.1 [6]

A pair (F, E) is called a soft set (over U) if and only if F is a mapping of E into the set of all subsets of the set U. In other words, the soft set is a parameterized family of subsets of the set U. Every set $F(\varepsilon),\ \varepsilon \in E$, from this family may be considered as the set of ε- elements of the soft set (F, E), or as the set of ε- approximate elements of the soft set.

Definition 1.1.2 [4]

A pair (F, A) is called a fuzzy soft set over U where $F : A \to \tilde{P}(U)$ is a mapping from A into $\tilde{P}(U)$. Here $\tilde{P}(U)$ represents the fuzzy subsets of U.

Definition 1.1.3 [1]

Let U be a universe and E a set of attributes. Then the pair (U, E) denotes the collection of all fuzzy soft sets on U with attributes from E and is called a fuzzy soft class.

Definition 1.1.4 [4]

A fuzzy soft set (F, A) over U is said to be null fuzzy soft set denoted by ϕ if $\forall \varepsilon \in A,\ F(\varepsilon)$ is the null fuzzy set $\overline{0}$ of U, where $\overline{0}(x) = 0\ \forall x \in U$.

We would use the notation (ϕ, A) to represent the fuzzy soft null set with respect to the set of parameters A.

Definition 1.1.5 [4]

A fuzzy soft set (F, A) over U is said to be absolute fuzzy soft set denoted by $\tilde{A}$ if $\forall \varepsilon \in A,\ F(\varepsilon)$ is the absolute fuzzy set $\bar{1}$ of U where $\bar{1}(x) = 1\ \forall x \in U$.

We would use the notation (U, A) to represent the fuzzy soft absolute set with respect to the set of parameters A.

Definition 1.1.6 [4]

For two fuzzy soft sets (F, A) and (G, B) in a fuzzy soft class (U, E), we say that (F, A) is a fuzzy soft subset of (G, B), if

(i) $A \subseteq B$

(ii) For all $\varepsilon \in A, F(\varepsilon) \subseteq G(\varepsilon)$ and is written as (F, A) $\tilde{\subseteq}$ (G, B).

Definition 1.1.7 [4]

Union of two fuzzy soft sets (F, A) and (G, B) in a soft class (U, E) is a fuzzy soft set (H, C) where $C = A \cup B$ and $\forall \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & \text{if } \varepsilon \in A - B \\ G(\varepsilon), & \text{if } \varepsilon \in B - A \\ F(\varepsilon) \cup G(\varepsilon), & \text{if } \varepsilon \in A \cap B \end{cases} \quad \text{and is written as } (F, A)\tilde{\cup}(G, B) = (H, C).$$

Definition 1.1.8 [1]

Let (F, A) and (G, B) be two fuzzy soft sets in a soft class (U, E) with $A \cap B \neq \phi$. Then Intersection of two fuzzy soft sets (F, A) and (G, B) in a soft class (U, E) is a fuzzy soft set (H, C) where $C = A \cap B$ and $\forall \varepsilon \in C$, $H(\varepsilon) = F(\varepsilon) \cap G(\varepsilon)$. We write $(F, A)\tilde{\cap}(G, B) = (H, C)$.

Definition 1.1.9 [7]

The complement of a fuzzy soft set (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$ where $F^c : A \to \tilde{P}(U)$ is a mapping given by $F^c(\alpha) = [F(\alpha)]^c,\ \forall \alpha \in A$.

II. FUZZY SOFT RELATIONS

Definition 2.1 (Product of Fuzzy Soft Sets)

Let (F, A) and (G, B) be two fuzzy soft sets over (U, E), where $U = \{c_1, c_2, c_3, \ldots\ldots\ldots\ldots c_m\}$ and $E = \{e_1, e_2, e_3, \ldots\ldots\ldots\ldots e_n\}$. Then

$(F, A) \times (G, B) = (H, C)$, where $C = A \times B$ and for any $(e_i, e_j) \in C,\ H(e_i, e_j) = F(e_i) \cap G(e_j)$ where $\cap$ is the intersection of the fuzzy sets $F(e_i)$ and $G(e_j)$.

Example 2.1

Let U = $\{s_1, s_2, s_3, s_4\}$ be the set of students of a school

E = {honest (e_1), sincere (e_2), punctual (e_3), talented (e_4), obedient (e_5)} be the set of parameters.

Let A= {e_1, e_2, e_3} $\subseteq$ E, B = {e_1, e_5} $\subseteq$ E

(F, A) = {F(e_1) = $\{s_1/0.5, s_2/0.4, s_3/0.7, s_4/0.9\}$, F($e_2$) = $\{s_1/0.3, s_2/0.6, s_3/0.2, s_4/0.7\}$,

F(e_3) = $\{s_1/0.5, s_2/0.8, s_3/0.7, s_4/0.3\}$ }

$(G, B) = \{G(e_1) = \{s_1/0.7, s_2/0.2, s_3/0.5, s_4/0.5\},\ G(e_5) = \{s_1/0.6, s_2/0.4, s_3/0.9, s_4/0.5\}\}$

Then (F, A) × (G, B) = (H, C), where C = A × B = {(e_1, e_1), (e_1, e_5), (e_2, e_1) , (e_2, e_5) , (e_3, e_1) , (e_3, e_5)}

and (H, C) $= \{H(e_1, e_1) = \{s_1/0.5, s_2/0.2, s_3/0.5, s_4/0.5\},\ H(e_1, e_5) = \{s_1/0.5, s_2/0.4, s_3/0.7, s_4/0.5\},$

$H(e_2, e_1) = \{s_1/0.3, s_2/0.2, s_3/0.2, s_4/0.5\},\ H(e_2, e_5) = \{s_1/0.3, s_2/0.4, s_3/0.2, s_4/0.5\},$

$H(e_3, e_1) = \{s_1/0.5, s_2/0.2, s_3/0.5, s_4/0.3\}, H(e_3, e_5) = \{s_1/0.5, s_2/0.4, s_3/0.7, s_4/0.3\}\}$

Definition 2.2 (Fuzzy Soft Relations)

Let (F, A) and (G, B) be two fuzzy soft sets over (U, E), where $U = \{c_1, c_2, c_3, \ldots\ldots\ldots\ldots c_m\}$ and $E = \{e_1, e_2, e_3, \ldots\ldots\ldots\ldots e_n\}$. Then any fuzzy soft subset of $(F, A) \times (G, B)$ is a fuzzy soft relation from (F, A) to (G, B).

A fuzzy soft relation from (F, A) to (F, A) is a fuzzy soft relation on (F, A).

Example 2.2
Let U = $\{s_1, s_2, s_3, s_4\}$ be the set of students of a school and
E = {honest (e_1), sincere (e_2), punctual (e_3), talented (e_4), obedient (e_5)} be the set of parameters.
Let A= {e_1, e_2, e_3} $\subseteq$ E, B = {e_1, e_5} $\subseteq$ E
(F, A) = {F(e_1) = $\{s_1/0.5, s_2/0.4, s_3/0.7, s_4/0.9\}$, F($e_2$) = $\{s_1/0.3, s_2/0.6, s_3/0.2, s_4/0.7\}$,
F(e_3) = $\{s_1/0.5, s_2/0.8, s_3/0.7, s_4/0.3\}$ }
$(G, B) = \{G(e_1) = \{s_1/0.7, s_2/0.2, s_3/0.5, s_4/0.5\}, G(e_5) = \{s_1/0.6, s_2/0.4, s_3/0.9, s_4/0.5\}\}$
We take C = {(e_1, e_1), (e_2, e_1), (e_2, e_5)} $\subseteq A \times B$
Let (R, C) = $\{R(e_1, e_1) = \{s_1/0.5, s_2/0.1, s_3/0.2, s_4/0.1\}, R(e_2, e_1) = \{s_1/0.2, s_2/0.2, s_3/0.1, s_4/0.4\},$
$R(e_2, e_5) = \{s_1/0.3, s_2/0.4, s_3/0.1, s_4/0.4\}\}$
Here (R, C) $\tilde{\subseteq}$ (F, A) $\times$ (G, B) and hence a fuzzy soft relation from (F, A) to (G, B).

Definition 2.3 (Fuzzy soft symmetric relation)
A fuzzy soft relation (R, C) on (F, A) is said to be fuzzy soft symmetric relation if, for each (e_i, e_j) $\in C \subseteq A \times A$, there exists ($e_j$, e_i) $\in C \subseteq A \times A$. It is obvious that R ($e_i$, e_j) = R (e_j, e_i), $\forall e_i, e_j \in A$.

Example 2.3
Let U = $\{s_1, s_2, s_3, s_4\}$ be the set of students of a school
E = {honest (e_1), sincere (e_2), punctual (e_3), talented (e_4), obedient (e_5)} be the set of parameters.
Let A= {e_1, e_2, e_3} $\subseteq$ E
(F, A) = {F(e_1) = $\{s_1/0.5, s_2/0.4, s_3/0.7, s_4/0.9\}$, F($e_2$) = $\{s_1/0.3, s_2/0.6, s_3/0.2, s_4/0.7\}$,
F(e_3) = $\{s_1/0.5, s_2/0.8, s_3/0.7, s_4/0.3\}$ }
We take C = {(e_1, e_1), (e_2, e_1), (e_1, e_2)} $\subseteq A \times A$
Let (R, C) = $\{R(e_1, e_1) = \{s_1/0.5, s_2/0.4, s_3/0.7, s_4/0.9\}, R(e_2, e_1) = \{s_1/0.3, s_2/0.4, s_3/0.2, s_4/0.7\},$
$R(e_1, e_2) = \{s_1/0.3, s_2/0.4, s_3/0.1, s_4/0.4\}\}$
Here (R, C) $\tilde{\subseteq}$ (F, A) $\times$ (F, A) and is a fuzzy soft symmetric relation on (F, A).

Definition 2.4 (Fuzzy soft inverse relation)
Let R be fuzzy soft relation from (F, A) to (G, B) then inverse relation R $^{-1}$ is defined as
R $^{-1}$ (e_i, e_j) = R (e_j, e_i), $\forall (e_i, e_j) \in D \subseteq B \times A$.

Example 2.4
Define a fuzzy soft inverse relation R $^{-1}$ from (G, B) to (F, A). From **Example 3.2**
We take D = {(e_1, e_1), (e_1, e_2), (e_5, e_2)} $\subseteq B \times A$
Then (R^{-1}, D) = $\{R^{-1}(e_1, e_1) = \{s_1/0.5, s_2/0.1, s_3/0.2, s_4/0.1\}, R^{-1}(e_1, e_2) = \{s_1/0.2, s_2/0.2, s_3/0.1, s_4/0.4\},$
$R^{-1}(e_5, e_2) = \{s_1/0.3, s_2/0.4, s_3/0.1, s_4/0.4\}\}$

Proposition 2.1
If R is fuzzy soft relation from (F, A) to (G, B) then R $^{-1}$ is a fuzzy soft relation from (G, B) to (F, A).

Proof
R $^{-1}$ (e_i, e_j) = R (e_j, e_i) $\subseteq$ F (e_j) $\cap$ G (e_i) = G (e_i) $\cap$ F (e_j), $\forall (e_i, e_j) \in C \subseteq B \times A$.

It follows that R $^{-1}$ is fuzzy soft relation from (G, B) to (F, A).

Definition 2.5 (Composition of Fuzzy Soft Relation)
Let R and S be two fuzzy soft relations from (F, A) to (G, B) and (G, B) to (H, C) respectively. Then the composition $\circ$ of R and S is defined by $(R \circ S)(e_i, e_j) = R(e_i, e_k) \cap S(e_k, e_j)$.

Example 2.5
Let U = $\{s_1, s_2, s_3, s_4\}$ be the set of students of a school
E = {honest (e_1), sincere (e_2), punctual (e_3), talented (e_4), obedient (e_5)} be the set of parameters.
Let A= {e_1, e_2, e_3} $\subseteq$ E, B = {e_1, e_5} $\subseteq$ E, C = {e_2} $\subseteq$ E
(F, A) = {F(e_1) = $\{s_1/0.5, s_2/0.4, s_3/0.7, s_4/0.9\}$, F($e_2$) = $\{s_1/0.3, s_2/0.6, s_3/0.2, s_4/0.7\}$,
F(e_3) = $\{s_1/0.5, s_2/0.8, s_3/0.7, s_4/0.3\}$ }
$(G,B) = \{G(e_1) = \{s_1/0.7, s_2/0.2, s_3/0.5, s_4/0.5\}, G(e_5) = \{s_1/0.6, s_2/0.4, s_3/0.9, s_4/0.5\}\}$
$(H,C) = \{H(e_2) = \{s_1/0.6, s_2/0.3, s_3/0.4, s_4/0.5\}\}$
Let P = {(e_1, e_5), (e_1, e_1)} $\subseteq A \times B$, Q = {(e_5, e_2)} $\subseteq B \times C$
(R, P) = $\{R(e_1,e_5) = \{s_1/0.5, s_2/0.4, s_3/0.7, s_4/0.5\}, R(e_1,e_1) = \{s_1/0.5, s_2/0.2, s_3/0.5, s_4/0.5\}\}$
And (S, Q) = $\{S(e_5,e_2) = \{s_1/0.6, s_2/0.3, s_3/0.4, s_4/0.5\}\}$
It follows that
(R $\circ$ S) (e_1, e_2) = R (e_1, e_5) $\cap$ S (e_5, e_2) = $\{s_1/0.5, s_2/0.3, s_3/0.4, s_4/0.5\}$

Proposition 2.2
If R and S be two fuzzy soft relations from (F, A) to (G, B) and (G, B) to (H, C) respectively, then R $\circ$ S is fuzzy soft relation from (F, A) to (H, C).
Proof
By definition
$R(e_i,e_j) = \{x/\min(\mu_{F(e_i)}, \mu_{G(e_j)}) : x \in U\}, \forall (e_i,e_j) \in A \times B$
$S(e_j,e_k) = \{x/\min(\mu_{G(e_j)}, \mu_{H(e_k)})\}, \forall (e_j,e_k) \in B \times C$
Therefore
$(R \circ S)(e_i,e_k) = R(e_i,e_j) \cap S(e_j,e_k)$
$= \{x/\min(\min(\mu_{F(e_i)}, \mu_{G(e_j)}), \min(\mu_{G(e_j)}, \mu_{H(e_k)})) : x \in U\}$
Now, $\min(\min(\mu_{F(e_i)}, \mu_{G(e_j)}), \min(\mu_{G(e_j)}, \mu_{H(e_k)})) = \min(\min(\mu_{F(e_i)}, \mu_{G(e_j)}, \mu_{H(e_k)}))$
$= \min(\mu_{F(e_i)}, \mu_{G(e_j)}, \mu_{H(e_k)})$
$\leq \min(\mu_{F(e_i)}, 1, \mu_{H(e_k)})$
$= \min(\mu_{F(e_i)}, \mu_{H(e_k)})$
It follows that $R(e_i,e_j) \cap S(e_j,e_k) \subseteq F(e_i) \cap H(e_k)$
Thus $R \circ S$ is fuzzy soft relation from (F, A) to (H, C).

Example 2.6
The composition operation of fuzzy soft mappings is not commutative.
Proof:
Let U = $\{s_1, s_2, s_3, s_4\}$ be the set of students of a school
E = {honest (e_1), sincere (e_2), punctual (e_3), talented (e_4), obedient (e_5)} be the set of parameters.
Let A= {e_1, e_2, e_3, e_5} $\subseteq$ E, B = {e_1, e_2, e_5} $\subseteq$ E, C = { e_1 , e_2 , e_5} $\subseteq$ E
(F, A) = {F(e_1) = $\{s_1/0.5, s_2/0.4, s_3/0.7, s_4/0.9\}$, F($e_2$) = $\{s_1/0.3, s_2/0.6, s_3/0.2, s_4/0.7\}$,
F(e_3) = $\{s_1/0.5, s_2/0.8, s_3/0.7, s_4/0.3\}$, F($e_5$) = $\{s_1/0.6, s_2/0.8, s_3/0.8, s_4/0.3\}$ }
$(G,B) = \{G(e_1) = \{s_1/0.7, s_2/0.2, s_3/0.5, s_4/0.5\}, \{G(e_2) = \{s_1/0.3, s_2/0.2, s_3/0.4, s_4/0.5\},$
$G(e_5) = \{s_1/0.6, s_2/0.4, s_3/0.9, s_4/0.5\}\}$
$(H,C) = \{H(e_1) = \{s_1/0.2, s_2/0.3, s_3/0.9, s_4/0.5\}, H(e_2) = \{s_1/0.6, s_2/0.3, s_3/0.4, s_4/0.5\},$
$H(e_5) = \{s_1/0.5, s_2/0.3, s_3/0.8, s_4/0.5\}\}$
Let P = {(e_1, e_5), (e_1, e_1), (e_5, e_2)} $\subseteq A \times B$, Q = {(e_5, e_2), (e_1, e_5)} $\subseteq B \times C$
(R, P) = $\{R(e_1,e_5) = \{s_1/0.5, s_2/0.4, s_3/0.7, s_4/0.5\}, R(e_1,e_1) = \{s_1/0.5, s_2/0.2, s_3/0.5, s_4/0.5\},$
$R(e_5,e_2) = \{s_1/0.3, s_2/0.2, s_3/0.1, s_4/0.2\}\}$
And (S, Q) = $\{S(e_5,e_2) = \{s_1/0.6, s_2/0.3, s_3/0.4, s_4/0.5\}, S(e_1,e_5) = \{s_1/0.4, s_2/0.2, s_3/0.4, s_4/0.5\}\}$
It follows that

(R $\circ$ S) (e_1, e_2) = R (e_1, e_5) $\cap$ S (e_5, e_2) = $\{s_1/0.5, s_2/0.3, s_3/0.4, s_4/0.5\}$

And (S $\circ$ R) (e_1, e_2) = S (e_1, e_5) $\cap$ R (e_5, e_2) = $\{s_1/0.3, s_2/0.2, s_3/0.1, s_4/0.2\}$

Consequently composition of fuzzy soft relations is not commutative.

Proposition 2.3

The composition of fuzzy soft relation is associative.

Proof

Let R, S, T be three fuzzy soft relations from (F, A) to (G, B), (G, B) to (H, C) and (H, C) to (L, D) respectively. Then

((R $\circ$ S) $\circ$ T) (e_1, e_4) = (R $\circ$ S) (e_1, e_3) $\cap$ T (e_3, e_4)

= (R (e_1, e_2) $\cap$ S (e_2, e_3)) $\cap$ T (e_3, e_4)

= R (e_1, e_2) $\cap$ (S (e_2, e_3)) $\cap$ T (e_3, e_4)

= R (e_1, e_2) $\cap$ (S $\circ$ T) (e_2, e_4)

= (R $\circ$ (S $\circ$ T)) (e_1, e_4)

Hence (R $\circ$ S) $\circ$ T = R $\circ$ (S $\circ$ T)

Definition 2.6 (Fuzzy Soft Reflexive Relation)

A fuzzy soft relation R on (F, A) is said to be fuzzy soft reflexive relation if, R (e_i, e_j) ⊆ R (e_i, e_i) and R (e_j, e_i) ⊆ R (e_i, e_i), $\forall e_i, e_j \in A$

Example 2.7

Let U = $\{s_1, s_2, s_3, s_4\}$ be the set of students of a school

E = {honest (e_1), sincere (e_2), punctual (e_3), talented (e_4), obedient (e_5)} be the set of parameters.

Let A= {e_1, e_2, e_3} $\subseteq$ E

(F, A) = {F (e_1) = $\{s_1/0.5, s_2/0.4, s_3/0.7, s_4/0.9\}$, F($e_2$) = $\{s_1/0.3, s_2/0.6, s_3/0.2, s_4/0.7\}$,

F(e_3) = $\{s_1/0.5, s_2/0.8, s_3/0.7, s_4/0.3\}$ }

We take C = {(e_1, e_1), (e_2, e_1), (e_2, e_2)} $\subseteq A \times A$

Let (R, C) = $\{R(e_1, e_1) = \{s_1/0.5, s_2/0.4, s_3/0.7, s_4/0.9\}$, $R(e_2, e_1) = \{s_1/0.3, s_2/0.4, s_3/0.1, s_4/0.2\}$,

$R(e_1, e_2) = \{s_1/0.3, s_2/0.4, s_3/0.1, s_4/0.4\}\}$

Here R (e_2, e_1) $\subseteq$ R (e_1, e_1), R (e_1, e_2) $\subseteq$ R (e_1, e_1) and R is a fuzzy soft reflexive relation on (F, A).

Definition 2.7 (Fuzzy Soft Transitive Relation)

A fuzzy soft relation R on (F, A) is said to be fuzzy soft transitive relation if R $\circ$ R $\tilde{\subseteq}$ R

Definition 2.8 (Fuzzy Soft Equivalence Relation)

If a fuzzy soft relation R on (F, A) is simultaneously reflexive, symmetric and transitive then it is known as a fuzzy soft equivalence relation.

Proposition 2.4

For a fuzzy soft relation R on (F, A), R = R^{-1} if and only if R is symmetric fuzzy soft relation on (F, A).

Proof

Let R = R^{-1}

R (e_i, e_j) = R^{-1} (e_j, e_i) = R (e_j, e_i)

∴ R is symmetric.

Conversely, Let R be a symmetric fuzzy soft relation.

R^{-1} (e_i, e_j) = R (e_j, e_i) = R (e_i, e_j)

∴ R = R^{-1}

Proposition 2.5

If R be a fuzzy soft equivalence relation on (F, A) then R^{-1} is also a fuzzy soft equivalence relation.

Proof

R^{-1} (e_i, e_j) = R (e_j, e_i) ⊆ R (e_i, e_i) = R^{-1} (e_i, e_i), since R is reflexive.

R^{-1} (e_j, e_i) = R (e_i, e_j) ⊆ R (e_i, e_i) = R^{-1} (e_i, e_i)

Thus R^{-1} is reflexive

R^{-1} (e_i, e_j) = R (e_j, e_i) = R (e_i, e_j) = R^{-1} (e_j, e_i), since R is symmetric

Thus R^{-1} is symmetric.

R^{-1} (e_i, e_j) = R (e_j, e_i) $\supseteq$ (R $\circ$ R) (e_j, e_i)

$= R(e_j, e_k) \cap R(e_k, e_i)$
$= R(e_k, e_j) \cap R(e_i, e_k)$
$= R^{-1}(e_j, e_k) \cap R^{-1}(e_k, e_i)$
$= R^{-1}(e_k, e_j) \cap R^{-1}(e_i, e_k)$ (since R^{-1} is symmetric)
$= R^{-1}(e_i, e_k) \cap R^{-1}(e_k, e_j)$
$= (R^{-1} \circ R^{-1})(e_j, e_i)$
$R^{-1} \circ R^{-1} \tilde{\subseteq} R^{-1}$

$\therefore R^{-1}$ is transitive and hence R^{-1} is a fuzzy soft equivalence relation.

III. SIMILARITY OF FUZZY SOFT SETS

Definition 3.1 (Scalar Cardinality of a fuzzy soft set)
Let (F, E) be a fuzzy soft sets over (U, E), where $U = \{c_1, c_2, c_3, \ldots\ldots\ldots c_m\}$ and $E = \{e_1, e_2, e_3, \ldots\ldots\ldots e_n\}$. The scalar cardinality of (F, E) is defined as
$|(F,E)| = \sum_j |F(e_j)|$, where $|F(e_j)|$ represent the scalar cardinality of each fuzzy set $F(e_j)$.

Definition 3.2 (Similarity Between Two Fuzzy Soft Sets)
Let (F, E) and (G, E) be two fuzzy soft sets over (U, E), where $U = \{c_1, c_2, c_3, \ldots\ldots\ldots c_m\}$ and $E = \{e_1, e_2, e_3, \ldots\ldots\ldots e_n\}$. Let $(F,E)\tilde{\cup}(G,E) = (P,E)$ and $(F,E)\tilde{\cap}(G,E) = (Q,E)$. We assume that $A = [a_{ij}]$ and $B = [b_{ij}]$ are the fuzzy soft matrices corresponding to the fuzzy soft sets (P, E) and (Q, E) respectively.
Let $M((F,E),(G,E)) \in [0,1]$ denote the similarity between the fuzzy soft sets (F, E) and (G, E).
We define $M((F,E),(G,E)) = \frac{|(Q,E)|}{|(P,E)|}$

Proposition 3.1
Let (F, E), (G, E) and (H, E) be three fuzzy soft sets over (U, E). Then the following results are valid.

(i) $M((F,E),(G,E)) = M((G,E),(F,E))$
(ii) $(F,E) = (G,E) \Rightarrow M((F,E),(G,E)) = 1$
(iii) $(F,E)\tilde{\cap}(G,E) = \tilde{\varphi} \Leftrightarrow M((F,E),(G,E)) = 0$
(iv) $(F,E) \tilde{\subseteq} (H,E) \tilde{\subseteq} (G,E) \Rightarrow M((F,E),(G,E)) \le M((H,E),(G,E))$

Proof:
(i) Let $(F,E)\tilde{\cup}(G,E) = (P,E)$ and $(F,E)\tilde{\cap}(G,E) = (Q,E)$. Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be the fuzzy soft matrices corresponding to the fuzzy soft sets (P, E) and (Q, E) respectively. Then $M((F,E),(G,E)) = \frac{|(Q,E)|}{|(P,E)|}$
Since $(F,E)\tilde{\cup}(G,E) = (G,E)\tilde{\cup}(F,E)$ and $(F,E)\tilde{\cap}(G,E) = (G,E)\tilde{\cap}(F,E)$, the proof is obvious.

(ii) Here $(F,E)\tilde{\cup}(G,E) = (F,E)\tilde{\cup}(F,E) = (F,E)$ and $(F,E)\tilde{\cap}(G,E) = (F,E)\tilde{\cap}(F,E) = (F,E)$.
respectively. Then $M((F,E),(G,E)) = \frac{|(F,E)|}{|(F,E)|} = 1$.
(iii) Here $(F,E)\tilde{\cap}(G,E) = \tilde{\varphi}$ so that $|(F,E)\tilde{\cap}(G,E)| = 0$ and hence $M((F,E),(G,E)) = 0$.

(iv) Let $(F,E) \tilde{\subseteq} (H,E) \tilde{\subseteq} (G,E)$
Then $\forall e \in E, F(e) \le H(e) \le G(e) \Rightarrow \forall e \in E, F(e) \cap G(e) \subseteq H(e) \cap G(e)$.
It follows that $M((F,E),(G,E)) \le M((H,E),(G,E))$

Example 3.1

Let $U = \{c_1, c_2, c_3\}$ be the set of three cars under consideration and E = $\{e_1$ (in good condition), e_2 (luxurious), e_3 (new technology) } be a set of parameters.

We consider two fuzzy soft sets (F, E) and (G, E) as

$(F, E) \quad = \{F(e_1) = \{(c_1, 0.2), (c_2, 0.3), (c_3, 0.4)\},\ F(e_2) = \{(c_1, 0.1), (c_2, 0.4), (c_3, 0.7)\},$

$F(e_3) = \{(c_1, 0.7), (c_2, 0.4), (c_3, 0.3)\}\}$

$(G, E) \quad = \{G(e_1) = \{(c_1, 0.4), (c_2, 0.1), (c_3, 0.6)\},\ G(e_2) = \{(c_1, 0.1), (c_2, 0.8), (c_3, 0.6)\},$

$G(e_3) = \{(c_1, 0.5), (c_2, 0.1), (c_3, 0.3)\}\}$

Let $(F, E) \tilde{\cup} (G, E) = (P, E)$ and $(F, E) \tilde{\cap} (G, E) = (Q, E)$.

We have then

$(P, E) \quad = \{P(e_1) = \{(c_1, 0.4), (c_2, 0.3), (c_3, 0.6)\},\ P(e_2) = \{(c_1, 0.1), (c_2, 0.8), (c_3, 0.7)\},$

$P(e_3) = \{(c_1, 0.7), (c_2, 0.4), (c_3, 0.3)\}\}$

$(Q, E) \quad = \{Q(e_1) = \{(c_1, 0.2), (c_2, 0.1), (c_3, 0.4)\},\ Q(e_2) = \{(c_1, 0.1), (c_2, 0.4), (c_3, 0.6)\},$

$Q(e_3) = \{(c_1, 0.5), (c_2, 0.1), (c_3, 0.3)\}\}$

The fuzzy soft matrices corresponding to (P, E) and (Q, E) are given by A and B are respectively,

$$A = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \end{array} \begin{array}{c} \begin{array}{ccc} e_1 & e_2 & e_3 \end{array} \\ \begin{bmatrix} 0.4 & 0.1 & 0.7 \\ 0.3 & 0.8 & 0.4 \\ 0.6 & 0.7 & 0.3 \end{bmatrix} \end{array} \quad \text{and} \quad B = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \end{array} \begin{array}{c} \begin{array}{ccc} e_1 & e_2 & e_3 \end{array} \\ \begin{bmatrix} 0.2 & 0.1 & 0.5 \\ 0.1 & 0.4 & 0.1 \\ 0.4 & 0.6 & 0.3 \end{bmatrix} \end{array}$$

We have, $|(P, E)| \quad = |A|$

$$= |P(e_1)| + |P(e_2)| + |P(e_3)|$$
$$= 1.3 + 1.6 + 1.4$$
$$= 4.3$$

$|(Q, E)| \quad = |B|$

$$= |Q(e_1)| + |Q(e_2)| + |Q(e_3)|$$
$$= 0.7 + 1.1 + 0.9$$
$$= 2.7$$

Thus we have

$$M((F, E), (G, E)) = \frac{|(Q, E)|}{|(P, E)|} = \frac{2.7}{4.3}$$

$= 0.6$

IV. APPLICATION IN A MEDICAL DIAGNOSIS

Definition 5.1 (Significantly Similar Fuzzy Soft Sets)

Let (F, E) and (G, E) be two fuzzy soft sets over the same soft universe (U, E). These two fuzzy soft sets will be called significantly similar if $M((F, E), (G, E)) \geq 0.5$

Illustration

Using the notion of similarity, we consider a hypothetical case study and try to find out whether a person having certain symptoms is suffering from malaria or not. With the help of a medical specialist, we first construct a model fuzzy soft set for malaria. Observing the ill person, we then form the fuzzy soft set of symptoms for the ill persons. Next we find the similarity of these two sets. If these two fuzzy soft sets are significantly similar then we conclude that the person is possibly suffering from malaria.

Let the universal set contain two elements 'suffer from (u_1)' and 'does not suffer from (u_2)', i.e. $U = \{u_1, u_2\}$. The set of parameters E is the set of symptoms.
Let E = {e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8}, where e_1 = Fever, e_2 = Chills, e_3 = Headache, e_4 = Nausea and Vomiting, e_5 = Muscle pain and fatigue, e_6 = Chest or abdominal pain, e_7 = Cough, e_8 = Sweating.
Our model fuzzy soft set for malaria (F,E) is given in **Table 1** and this can be prepared with the help of a medical specialist. In a similar way, we form the fuzzy soft sets corresponding to the two ill persons under consideration as given in **Table 2, 3** respectively.

Table 1 (Model Fuzzy Soft Set for Malaria)

(F,E)	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
u_1	1	1	1	1	1	0.8	0.7	0.8
u_2	0	0	0	0	0	0.2	0.3	0.2

Table 2 (Fuzzy Soft Set for 1st ill person)

(F_1,E)	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
u_1	0.2	0.1	0.3	0.3	0.9	0.4	0.4	0.2
u_2	0.1	0.3	0.2	0.5	0.6	0.5	0.3	0.7

Table 3 (Fuzzy Soft Set for 2nd ill person)

(F_2,E)	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
u_1	0.4	0.3	0.5	0.7	0.8	0.3	0.7	0.5
u_2	0.5	0.4	0.2	0.4	0.6	0.6	0.2	0.4

Case I (Similarity between (F,E) and (F_1,E))

$$(F,E)\tilde{\cup}(F_1,E)=(P_1,E)$$

(P_1,E)	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
u_1	1	1	1	1	1	0.8	0.7	0.8
u_2	0.1	0.3	0.2	0.5	0.6	0.5	0.3	0.7

$$(F,E)\tilde{\cap}(F_1,E)=(Q_1,E)$$

(Q_1,E)	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
u_1	0.2	0.1	0.3	0.3	0.9	0.4	0.4	0.2
u_2	0	0	0	0	0	0.2	0.3	0.2

The fuzzy soft matrices corresponding to these two fuzzy soft sets (P_1,E) and (Q_1,E) are given by,

$$A_1 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0.8 & 0.7 & 0.8 \\ 0.1 & 0.3 & 0.2 & 0.5 & 0.6 & 0.5 & 0.3 & 0.7 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0.2 & 0.1 & 0.3 & 0.3 & 0.9 & 0.4 & 0.4 & 0.2 \\ 0 & 0 & 0 & 0 & 0 & 0.2 & 0.3 & 0.2 \end{bmatrix}$$

We have, $|(P_1,E)| = |A_1|$

$$=|P_1(e_1)|+|P_1(e_2)|+|P_1(e_3)|+|P_1(e_4)|+|P_1(e_5)|+|P_1(e_6)|+|P_1(e_7)|+|P_1(e_8)|$$

$$=1.1+1.3+1.2+1.5+1.6+1.3+1.0+1.5$$

$$=10.5$$

$$|(Q_1,E)| = |B_1|$$

$$=|Q_1(e_1)|+|Q_1(e_2)|+|Q_1(e_3)|+|Q_1(e_4)|+|Q_1(e_5)|+|Q_1(e_6)|+|Q_1(e_7)|+|Q_1(e_8)|$$

$$=0.2+0.1+0.3+0.3+0.9+0.6+0.7+0.4$$

$$=3.5$$

Thus we have

$$M((F,E),(F_1,E)) = \frac{|(Q_1,E)|}{|(P_1,E)|}$$

$$= \frac{3.5}{10.5}$$

$$\approx 0.3$$

Case II (Similarity between (F,E) and (F_2,E))

$$(F,E)\tilde{\cup}(F_2,E)=(P_2,E)$$

(P_2,E)	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
u_1	1	1	1	1	1	0.8	0.7	0.8
u_2	0.5	0.4	0.2	0.4	0.6	0.6	0.3	0.4

$$(F,E)\tilde{\cap}(F_2,E)=(Q_2,E)$$

(Q_2,E)	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
u_1	0.4	0.3	0.5	0.7	0.8	0.3	0.7	0.5
u_2	0	0	0	0	0	0.2	0.2	0.2

The fuzzy soft matrices corresponding to these two fuzzy soft sets (P_2,E) and (Q_2,E) are given by,

$$A_2 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0.8 & 0.7 & 0.8 \\ 0.5 & 0.4 & 0.2 & 0.4 & 0.6 & 0.6 & 0.3 & 0.4 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0.4 & 0.3 & 0.5 & 0.7 & 0.8 & 0.3 & 0.7 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 0.2 & 0.2 & 0.2 \end{bmatrix}$$

Thus we have

$$|(P_2,E)| = |A_1|$$

$$=|P_2(e_1)|+|P_2(e_2)|+|P_2(e_3)|+|P_2(e_4)|+|P_2(e_5)|+|P_2(e_6)|+|P_2(e_7)|+|P_2(e_8)|$$

$$=1.5+1.4+1.2+1.4+1.6+1.4+1.0+1.2$$

$$=10.7$$

$$|(Q_2,E)| = |B_2|$$

$$=|Q_2(e_1)|+|Q_2(e_2)|+|Q_2(e_3)|+|Q_2(e_4)|+|Q_2(e_5)|+|Q_2(e_6)|+|Q_2(e_7)|+|Q_2(e_8)|$$

$$=0.4+0.3+0.5+0.7+0.8+0.5+0.9+0.7$$

$$=4.8$$

$$M((F,E),(F_2,E)) = \frac{|(Q_2,E)|}{|(P_2,E)|}$$

$$= \frac{4.8}{10.7}$$

$$\approx 0.5$$

In view of our discussion, we can conclude that the second person has more possibility of suffering from malaria.

V. CONCLUSION

In our work, we have put forward some new notions of relations on fuzzy soft sets and similarity between two fuzzy soft sets. Some related properties have been established with proof and examples. A decision problem has been considered to get the optimal solution with the help of similarity of fuzzy soft sets. We hope that our work would enhance this study on fuzzy soft sets.

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