International Journal of Modern Engineering Research (IJMER)
www.ijmer.com Vol.1, Issue1, pp-25-38 ISSN: 2249-6645

EFFECT OF COUPLE-STRESS ON THE REFLECTION AND TRANSMISSION OF PLANE WAVES AT AN INTERFACE

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ABSTRACT

The present paper is concerned with the reflection and transmission of plane waves at an interface between two dissimilar couple-stress elastic half-spaces in perfect contact. Amplitude ratios of various reflected and refracted waves have been calculated and computed numerically for a specific model. The variations of these amplitude ratios with the angle of incidence have been shown graphically. The results of Graff and Pao [3] have been obtained as a special case and are shown graphically. The amplitude ratios at elastic-elastic interface have also been deduced.

Key Words: Couple–stress, elastic media, Amplitude ratios, Angle of incident, Welded contact, Stress free boundary.

INTRODUCTION

In the development of classical theory of elasticity the potential energy density is assumed to be a function of strain components, which are formed by a linear combination of the first order space derivatives of the displacements. The effects of surface moment per unit area have been neglected in the theory of elasticity [4] ; and only the surface force per-unit area, known as traction, has been taken into account. The components of traction are the force-stresses, extensions of the classical theory has been made to include the effects of higher-order derivatives of displacement in the potential energy. The work done by surface couples, surface forces and higher order stress like quantities have been included in the balance of energy in the new developments.

The development in the field of couple-stresses was mainly done by Mindlin and Tiersten [5], they developed a linear theory with a complete set of equations of motion, constitutive equations and boundary conditions for unique solutions. Aggarwal and Alverson [1]studied the effect of couple-stresses on diffraction of plane elastic waves by cylindrical discontinuities.

Sengupta and Ghosh [6] studied the effect of couple-stresses on the surface waves in elastic media, they deduced the equations of surface waves in elastic media under the influence of couple stresses and observed that the effect of couple-stresses increases the velocity of Rayleigh and Love wave propagation. Sengupta and Benerji [7] investigated the effects of couple-stresses on propagation of waves in elastic layer immersed in an infinite liquid. Graff and Pao [3] discussed the effects of couple-stresses on the propagation and reflection of plane waves in an elastic half space and obtained theoretically the amplitude ratios of various reflected waves.Recently Singh and Kumar [8] studied the reflection and refraction of plane waves at an interface between micropolar elastic solid and viscoelastic solid. Kumar and Singh [9] investigated the reflection of plane waves at a planer viscoelastic micropolar interface. Singh and Kumar [10] discussed the wave propagation in a micropolar elastic solid with stretch.In the present investigation, the problem of reflection and transmission of plane wave at an interface between two dissimilar couple-stress elastic solids in welded contact has been discussed. The results of Graff and Pao [3] have been deduced as a special case and are presented graphically.

BASIC EQUATION AND THEIR SOLUTIONS

The equation of motions without body forces and body couples and the constitutive relations in couple stress theory of elasticity are given by Mindlin, and Tiersten (1962) as

$$\mu\nabla^2\vec{u} + (\lambda + \mu)\ \text{grad div}\,\vec{u} + \eta\ \nabla^2\ (\text{grad div}\,\vec{u}) - \eta\nabla^2(\nabla^2\vec{u}) = \rho\frac{\partial^2\vec{u}}{\partial t^2}, \tag{1}$$

$$T_{ji} = T^{s}_{ji} + \frac{1}{2}\in_{jli}\ (\mu^{D}_{pl,p} + \mu_{,l}), \tag{2}$$

with

$$T^{s}_{ji} = \lambda \in_{kk} \delta_{ij} + 2\ \mu \in_{ij} \tag{3}$$

$$\mu_{ij} = \mu^{D}_{ij} + \mu\ \delta_{ij} \tag{4}$$

where

$$\mu^{D}_{ij} = 4\eta\ \Psi_{ij} + 4\eta'\ \Psi_{ji} \tag{5}$$

$$\in_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$$

$$\psi_{ij} = \omega_{j,i} = \frac{1}{2}\in_{jkl}\ u_{l,ki} \tag{6}$$

and

$$\mu^{D}_{ij} = \mu_{ij} - \mu\delta_{ij}.$$

The list of symbols is given at the end of the paper.

By Helmholtz representation of displacement vector, we can write

$$\vec{u} = \nabla\phi + \text{curl}\ \vec{\psi} \qquad \nabla.\vec{\psi} = 0 \tag{7}$$

Making use of equation (4) in equation (1), we get

$$\nabla^2\phi = \frac{1}{C_1^2}\frac{\partial^2\phi}{\partial t^2} \tag{8}$$

and

$$\nabla^2 (1 - l^2 \nabla^2) \quad \psi = \frac{1}{C_2^2}\frac{\partial^2\psi}{\partial t^2} \tag{9}$$

where

$$C_1{}^2 = \frac{\lambda + 2\mu}{\rho}, \qquad C_2{}^2 = \frac{\mu}{\rho}, \qquad l^2 = \frac{\eta}{\mu}$$

where ψ is the zth component of $\vec{\psi}$ and $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$

We are considering two dimensional problems in xy-plane, therefore the displacement and rotation vectors are taken as

$$\vec{u} = (u_x, u_y, 0)$$

$$\vec{w} = (0, 0, w_z) \tag{10}$$

Assuming time harmonic variation as exp(−iωt) in equation(5)and (6),we obtain

$$\left(\nabla^2 + \delta_1^2\right)\phi = 0 \tag{11}$$

$$\left(-\ell^2\nabla^4 + \nabla^2 + \delta_2^2\right)\psi = 0, \tag{12}$$

where $\delta_1^2 = \frac{\omega^2}{C_1^2} \quad \delta_2^2 = \frac{\omega^2}{C_2^2}$

To solve equation(11) and (12),and we assume the solution of equation (12) as**,**

$$\psi = \psi_1 + \psi_2$$

where ψ_1 and ψ_2 satisfy the following equations

$$(\nabla^2 + \delta_2^2)\psi_1 = 0, \qquad (\nabla^2 + \delta_3^2)\psi_2 = 0$$

$$\phi = f(y)\, e^{i(\xi x - \omega t)} \tag{10}$$

$$\psi = g(y)\, e^{i(\zeta x - \omega t)} \tag{11}$$

where ξ and ζ are the wave numbers. Substitution of (10) and (11) into (8) and (9) ,Yield second order and fourth order differential equation and the fourth order differential equation can be reduced to, two second order differential by letting

$$\psi(y) = \psi_1(y) + \psi_2(y) \tag{12}$$

Then the equations are

$$\frac{d^2f}{dy^2}+\alpha^2 f=0 \qquad (13)$$

$$\frac{d^2\psi_1}{dy^2}+\beta^2\psi_1=0 \qquad (14)$$

$$\frac{d^2\psi_2}{dy^2}-\gamma^2\psi_2=0 \qquad (15)$$

where

$$\alpha^2=\frac{\omega^2}{c_1^2}-\xi^2 \qquad (16)$$

$$\beta^2=\beta_1^2-\zeta^2 \qquad (17)$$

$$\gamma^2=\beta_2^2+\zeta^2 \qquad (18)$$

and

$$\beta_1{}^2=\left\{\ 1+\sqrt{1+\frac{4\ell^2\omega^2}{C_2^2}}\ \right\}/\,2\,\ell^2,$$

$$\beta_2{}^2=\left\{\ 1-\sqrt{1+\frac{4\ell^2\omega^2}{C_2^2}}\ \right\}/\,2\,\ell^2 \qquad (19)$$

From equations (16)-(18), it is seen that γ^2 is always greater than zero, where as α^2 and β^2 may have real value greater than equal to or less than zero .The solutions of (13) and (14) will of course differ accordingly.

Therefore, in an unbounded couple stress medium three types of waves propagate. In addition to P and SV-wave, there is SS-wave which decays rapidly from the surface for any reasonable value of the couple-stress material constant.

The stresses in terms of potentials are

$$T_{xy}=\mu\left[\frac{2\,\partial^2\phi}{\partial x\,\partial y}+\frac{\partial^2\psi}{\partial y^2}-\frac{\partial^2\psi}{\partial x^2}-l^2\nabla^4\psi\right], \qquad (20)$$

$$T_{yy}=\mu\left[K_1^2\nabla^2\phi-\frac{2\partial^2\phi}{\partial x^2}-\frac{2\partial^2\psi}{\partial x\,\partial y}\right], \qquad (21)$$

$$\mu_{yz}=-2\eta\frac{\partial}{\partial y}(\nabla^2\psi), \qquad (22)$$

where

$$K_1^2 = C_1^2 / C_2^2$$

III.FORMULATION OF THE PROBLEM

We consider two homogeneous couple-stress elastic half spaces M and M′ which are in welded contact at the interface y = 0. We take y-axis vertically downward and x-axis horizontally. We consider a plane harmonic body wave (P or Sv-wave) with time dependence proportional to exp(−iωt) propagating through the couple stress medium M which we identify as the region y > 0, and incident at the interface y = 0, with its direction of propagation making an angle θ_0 with the normal to surface. Corresponding to each incident wave, we get waves in couple stress medium M as reflected P and Sv-wave and the SS-wave whose amplitude decays exponentially from the surface and its wave front is at right angles to the surface; and refracted P-and Sv-waves and the SS-wave (traveling with its wave front at right angles to the surface and amplitude decays exponentially from the surface) transmitted to the couple-stress medium M′ as shown in fig. 1.

In the region y > 0, we write all the variables without a prime and we attach a prime to denote the variables in the region y < 0.

The appropriate potentials for this problem are given by

In medium M

$$\phi = A_1\, e^{i[\gamma_1 (x \sin\theta_0 - y\cos\theta_0) - \omega t]} + A_2\, e^{i[\gamma_1 (x\sin\theta_1 + y\cos\theta_1) - \omega t]} \tag{23}$$

$$\psi = B_1\, e^{i[\gamma_2 (x\sin\theta_0 - y\cos\theta_0) - \omega t]} + B_2\, e^{i[\gamma_2 (x\sin\theta_2 + y\cos\theta_2) - \omega t]} + C_2\, e^{[-\gamma y + i(x\gamma_2 \sin\theta_2 - \omega t)]} \tag{24}$$

where A_1, B_1, A_2, B_2, C_2 are amplitude of various incident and reflected waves.

In medium M′

$$\phi' = A_3 e^{i[\gamma_3 (x\sin\theta_3 - y\cos\theta_3) - \omega t]} \tag{25}$$

$$\psi' = B_3 e^{i[\gamma_4 (x\sin\theta_4 - y\cos\theta_4) - \omega t]} + C_3\, e^{[\gamma' y + i(x\gamma_4 \sin\theta_4 - \omega t)]} \tag{26}$$

where A_3, B_3, C_3 are amplitude of various refracted waves. Also B1 = 0 for incident P-wave, $A_1 = 0$ for incident Sv-wave. The Snell's Law is given by

$$\frac{\sin\theta_0}{C_1^*} = \frac{\sin\theta_1}{C_1} = \frac{\sin\theta_2}{C_s} = \frac{\sin\theta_3}{C_1'} = \frac{\sin\theta_4}{C_s'} \tag{27}$$

and

$$\gamma_1 C_1 = \gamma_2 C_s = \gamma_3 C_1' = \gamma_4 C_s' = \omega$$

$C_1^* = C_1$, for incident P-wave

$= C_s$, for incident Sv-wave

and

$$C_s^2 = C_2^2\,[1 + l^2\,\beta_1^2]$$ [Mindlin and Tiersten (1962)]

BOUNDARY CONDITIONS

The boundary conditions at the interface y = 0, where the two medium are assumed to be in welded contact are the continuity of tangential displacement, normal displacement, tangential rotational vector, normal force-stress, tangential force-stress and tangential couple-stress, i.e. at y = 0.

$$u_x = u'_x,\ u_y = u'_y,\ w_z = w'_z,\ T_{yy} = T'_{yy},\ T_{xy} = T'_{xy},\ \mu_{yz} = \mu'_{yz} \qquad (28)$$

Making use of potentials given by equations (23)-(26) in the boundary conditions (28) and with the help of equations (2),(3),(7) and (26), we obtain the six non-homogeneous equations which can be written as

$$\sum_{j=1}^{6} a_{ij}\, y_j = x_i \qquad (i = 1, 2,\ldots,6) \qquad (29)$$

where

$$a_{11} = \frac{i\gamma_1}{\gamma^*}\sin\theta_1, \quad a_{12} = \frac{i\gamma_2}{\gamma^*}\cos\theta_2, \quad a_{13} = \frac{-\gamma}{\gamma^*},$$

$$a_{14} = \frac{-i\gamma_3}{\gamma^*}\sin\theta_3, \quad a_{15} = \frac{i\gamma_4}{\gamma^*}\cos\theta_4, \quad a_{16} = \frac{-\gamma'}{\gamma^*}$$

$$a_{21} = \frac{i\gamma_1}{\gamma^*}\cos\theta_1, \quad a_{22} = \frac{-i\gamma_2\sin\theta_2}{\gamma^*}, \quad a_{23} = \frac{-i\gamma_2}{\gamma^*}\sin\theta_2,$$

$$a_{24} = \frac{i\gamma_3\cos\theta_3}{\gamma^*}, \quad a_{25} = \frac{i\gamma_4}{\gamma^*}\sin\theta_4, \quad a_{26} = \frac{i\gamma_4\sin\theta_4}{\gamma^*}$$

$$a_{32} = \frac{-\gamma_2^2}{\gamma^{*2}} \qquad a_{33} = \frac{(\gamma^2 - \gamma_2^2\sin^2\theta_2)}{\gamma^{*2}}$$

$$a_{35} = \frac{\gamma_4^2}{\gamma^{*2}}, \qquad a_{36} = \frac{-1(\gamma'^2 - \gamma_4^2\sin^2\theta_4)}{\gamma^{*2}}$$

$$a_{41} = \frac{\mu\gamma_1^2}{\gamma^{*2}}(2\sin^2\theta_1 - K^2), \qquad a_{42} = \frac{\mu\gamma_2^2}{\gamma^{*2}}\sin 2\theta_2$$

$$a_{43} = \frac{2i\mu\gamma\gamma_2}{\gamma^{*2}}\sin\theta_2, \qquad a_{44} = \frac{-\mu'\gamma_3^2}{\gamma^{*2}}(2\sin^2\theta_3 - K'^2)$$

$$a_{45} = \frac{\mu'\gamma_4^2}{\gamma^{*2}}\sin 2\theta_4, \qquad a_{46} = \frac{2i\mu'\gamma'\gamma_4}{\gamma^{*2}}\sin\theta_4$$

$$a_{51} = \frac{2\mu\gamma_1^2}{\gamma^{*2}}\sin\theta_1\cos\theta_1, \qquad a_{52} = \frac{-\mu}{\gamma^{*2}}\left(2\gamma_2^2\sin^2\theta_2 - \frac{\omega^2}{C_2^2}\right)$$

$$a_{53} = \frac{-\mu}{\gamma^{*2}}\left(2\gamma_2^2 \sin^2\theta_2 - \frac{\omega^2}{C_2^2}\right), \qquad a_{54} = \frac{2\mu'\gamma_3^2}{\gamma^{*2}} \sin\theta_3 \cos\theta_3$$

$$a_{55} = \frac{\mu'}{\gamma^{*2}}\left(2\gamma_4^2 \sin^2\theta_4 - \frac{\omega^2}{C_2'^2}\right) \qquad a_{56} = \frac{\mu'}{\gamma^{*2}}\left(2\gamma_4^2 \sin^2\theta_4 - \frac{\omega^2}{C_2'^2}\right)$$

$$a_{62} = i\eta \frac{\gamma_2^3}{\gamma^{*3}} \cos\theta_3 \qquad a_{63} = \frac{\eta}{\gamma^{*3}}(\gamma^3 - \gamma\gamma_2^2 \sin^2\theta_2)$$

$$a_{65} = \frac{i\eta'}{\gamma^{*3}}\gamma_4^3 \cos\theta_4 \qquad a_{66} = \frac{\eta'}{\gamma^{*3}}\left(\gamma'^3 - \gamma'\gamma_4^2 \sin^2\theta_4\right)$$

$$a_{31} = a_{34} = a_{61} = a_{64} = 0 \tag{30}$$

where

$$\gamma^* = \gamma_1 \text{ for incident P-wave.}$$
$$= \gamma_2 \text{ for incident sv-wave.} \tag{31}$$

(a) For incident P-wave.

$$x_1 = -a_{11},\ x_2 = a_{21},\ x_3 = a_{31},\ x_4 = -a_{41},\ x_5 = a_{51},\ x_6 = a_{61}.$$

(b) For incident Sv-wave.

$$x_1 = a_{12},\ x_2 = -a_{22},\ x_3 = -a_{32},\ x_4 = a_{42},\ x_5 = -a_{52},\ x_6 = a_{62}$$

The amplitude ratios of reflected and transmitted wave are,

$$y_1 = \frac{A_2}{A^*},\ y_2 = \frac{B_2}{A^*},\ y_3 = \frac{C_2}{A^*},\ y_4 = \frac{A_3}{A^*},\ y_5 = \frac{B_3}{A^*},\ y_6 = \frac{C_3}{A^*}$$

where

$$A^* = A_1, \text{ for incident P-wave}$$
$$= B_1, \text{ for incident Sv-wave.} \tag{32}$$

SPECIAL CASES

CASE I. If all the elastic moduli in medium M′ vanish, then the problem reduces to reflection of plane waves at a flat free boundary y = 0, in this case, the boundary conditions reduce to

$$T_{yy} = T_{xy} = \mu_{yz} = 0.$$

Therefore, from equation (29), we obtain

(a) For incident P-Wave

$$\frac{A_2}{A_1} = \left[\gamma\gamma_1^2\gamma_2^2 \sin 2\theta_1 \sin 2\theta_2 (\gamma^2 + \gamma_2^2 \cos^2\theta_2)\right.$$

$$-\left(2\gamma_2^2\sin^2\theta_2-\frac{\omega^2}{C_2^2}\right)\left(2\gamma_1^2\sin^2\theta_1-\frac{\omega^2}{C_2^2}\right)\left(\gamma^3-\gamma\gamma_2^2\sin^2\theta_2-1\gamma_2^3\cos\theta_2\right)\Big]\Big/D_1$$

$$\frac{B_2}{A_1}=\left[-2\gamma_1^2\sin 2\theta_1(\gamma^3-\gamma\,\gamma_2^2\sin^2\theta_2)\left(2\gamma_1^2\sin^2\theta_1-\frac{\omega^2}{C_2^2}\right)\right]\Big/D_1$$

$$\frac{C_2}{A_1}=\left[-2i\gamma^2\,\gamma_2^3\cos\theta_2\sin 2\theta_1\left(2\gamma_1^2\sin^2\theta_1-\frac{\omega^2}{C_2^2}\right)\right]\Big/D_1$$

where

$$D_1=\gamma\,\gamma_1^2\,\gamma_2^2\sin 2\theta_1\sin 2\theta_2\,(\gamma^2+\gamma_2^2\cos^2\theta_2)$$

$$+\left(2\gamma_2^2\sin^2\theta_2-\frac{\omega^2}{C_2^2}\right)\left(2\gamma_1^2\sin^2\theta_1-\frac{\omega^2}{C_2^2}\right)$$

$$(\gamma^3-\gamma\,\gamma_2^2\sin^2\theta_2-1\,\gamma_2^3\cos\theta_2)$$

(b) For incident Sv-wave

$$\frac{A_2}{B_1}=\left[(2\gamma_2^2\sin 2\theta_2)\left(2\gamma_2^2\sin^2\theta_2-\frac{\omega^2}{C_2^2}\right)\left(\gamma\gamma_2^2\cos^2\theta_2+\gamma^3\right)\right]\Big/D_2$$

$$\frac{B_2}{B_1}=\Big[\ (\gamma_1^2\sin 2\theta_1)(\gamma_2^2\sin 2\theta_2)(\gamma\gamma_2^2\cos^2\theta_2+\gamma^3)$$

$$-\left(2\gamma_2^2\sin^2\theta_2-\frac{\omega^2}{C_2^2}\right)\left(2\gamma_1^2\sin^2\theta_1-\frac{\omega^2}{C_2^2}\right)(\gamma^3-\gamma\gamma_2^2\sin^2\theta_2+i\gamma_2^3\cos\theta_2)\Big]\Big/D_2$$

$$\frac{C_2}{B_1}=\left[2i\gamma_2^3\cos\theta_2\left(2\gamma_1^2\sin^2\theta_1-\frac{\omega^2}{C_2^2}\right)\left(2\gamma_2^2\sin^2\theta_2-\frac{\omega^2}{C_2^2}\right)\right]\Big/D_2$$

where

$$D_2=(\gamma_1^2\sin 2\theta_1)\,(\gamma_2^2\sin 2\theta_2)(\gamma\gamma_2^2\cos^2\theta_2+\gamma^3)$$

$$+\left(2\gamma_2^2\sin^2\theta-\frac{\omega^2}{C_2^2}\right)\left(2\gamma_1^2\sin^2\theta_1-\frac{\omega^2}{C_2^2}\right)(\gamma^3-\gamma\gamma_2^2\sin^2\theta_2+i\gamma_2^3\cos\theta_2)$$

The above amplitude ratios are same as obtained by Graff and Pao [3].

CASE II. If we let $\eta=\eta'=0$, then both media are reduced to the isotropic elastic half spaces in welded contact and we obtain a set of four non-homogeneous equations as

$$\sum_{j=1}^{4} C_{ij}\, y_i = x_i \qquad (i = 1, 2, 3, 4) \qquad (33)$$

$C_{11} = \dfrac{i\gamma_1}{\gamma^*}\sin\theta_1$, $C_{12} = \dfrac{i\gamma_2}{\gamma^*}\cos\theta_2$, $C_{13} = \dfrac{-i\gamma_3}{\gamma^*}\sin\theta_3$

$C_{14} = \dfrac{i\gamma_4}{\gamma^*}\cos\theta_4$, $C_{21} = \dfrac{i\gamma_1}{\gamma^*}\cos\theta_1$, $C_{22} = \dfrac{-i\gamma_2\sin\theta_2}{\gamma^*}$

$C_{23} = \dfrac{i\gamma_3}{\gamma^*}\cos\theta_3$, $C_{24} = \dfrac{i\gamma_4\sin\theta_4}{\gamma^*}$

$C_{31} = \dfrac{\mu\gamma_1^2}{\gamma^{*2}}(2\sin^2\theta_1 - K_1^2)$, $C_{32} = \dfrac{\mu\gamma_2^2}{\gamma^{*2}}\sin 2\theta_2$

$C_{33} = \dfrac{-\mu'\gamma_3^2}{\gamma^{*2}}(2\sin^2\theta_3 - K_1'^2)$, $C_{34} = \dfrac{\mu'\gamma_4^2}{\gamma^{*2}}\sin 2\theta_4$

$C_{41} = \dfrac{2\mu\gamma_1^2}{\gamma^{*2}}\sin\theta_1\cos\theta_1$, $C_{42} = \dfrac{-\mu}{\gamma^{*2}}\left(2\gamma_2^2\sin^2\theta_2 - \dfrac{\omega^2}{C_2^2}\right)$

$C_{43} = \dfrac{2\mu'\gamma_3^2}{\gamma^{*2}}\sin\theta_3\cos\theta_3$, $C_{44} = \dfrac{\mu'}{\gamma^{*2}}\left(2\gamma_4^2\sin^2\theta_4 - \dfrac{\omega^2}{C_2'^2}\right)$

(c) For incident p-wave.

$x_1 = -C_{11},\ x_2 = C_{21},\ x_3 = -C_{31},\ x_4 = C_{41}$

(d) For incident Sv-wave.

$x_1 = C_{12},\ x_2 = -C_{22},\ x_3 = C_{32},\ x_4 = -C_{42}$

and

$$y_1 = \frac{A_2}{A^*},\ y_2 = \frac{B_2}{A^*},\ y_3 = \frac{A_3}{A^*},\ y_4 = \frac{B_3}{A^*}$$

where γ^* and A^* and given by equations (31) and (32) respectively.

The above results are same as obtained in Ewing, Jardetzky and Press [2].

NUMERICAL DISCUSSION

To discus the problem numerically when P-wave and Sv-wave are incident, we consider the following values of relevant parameters for couple-stress elastic solid.

For medium M ; (y > 0)

$\lambda = 2.2.$ dyne/cm^2, $\mu = 0.81$ dyne/cm^2, $\rho = 2.6$ gm/cm^3

For medium M′ (y < 0)

$\lambda' = 0.96$ dyne/cm^2, $\mu' = 0.71$ dyne/cm^2, $\rho' = 1.93$ gm/cm^3

The non-dimensional constants are $l^2\gamma_2^2 = 0.1,\ l'^2\gamma_4^2$ = 0.01 and η′/η = 0.05.

For the above values of relevant constants, the system of equations (29) is solved for amplitude ratios by using Gauss elimination method for different angle of incidence varying from θ^0 to 90^0. The variations of these amplitude ratios with the angle of incidence for incident P-wave and Sv-wave have been shown graphically in Fig. (2)-(19). The solid lines in these figures correspond to the variation of amplitude ratios in couple-stress elastic medium, whereas the dashed lines correspond to the variation of amplitude ratios for elastic medium.

CASE I. Couple-stress elastic and couple-stress elastic interface

Subcase (a) : Incident P-wave.

The comparison of amplitude ratio $|z_1|$ of reflected P-wave in couple stress elastic medium, with amplitude ratio $|z_1|$ of reflected P-wave in elastic medium shows that the effect of couple-stress increases the value of amplitude ratio $|z_1|$ of elastic medium with increase in angle of incidence. There is a sharp difference between the values of these amplitude ratios initially, but as the angle of incidence increases the difference decreases slowly and at $\theta_0 = 90^0$ the both amplitude ratios approaches the same value, these variation are shown in figure(2).

Fig. 3 depicts the variations of the reflection coefficients $|z_2|$ of the reflected Sv-wave in couple stress elastic and in elastic medium. The amplitude ratio $|z_2|$ of reflected Sv-wave in couple-stress elastic medium increases more rapidly than the amplitude ratio $|z_2|$ of reflected Sv-wave in elastic medium however, there is a sharp difference between the max. value of both amplitude ratios.

When we compare the amplitude ratio $|z_4|$ of refracted P-wave in couple-stress medium with amplitude ratio $|z_4|$ of refracted P-wave in elastic medium, we observe that both amplitude ratios decrease monotonically and approaches to same value at $\theta_0 = 90^0$, though the initial value of the amplitude ratio $|z_4|$ of couple-stress medium is more than the amplitude ratio $|z_4|$ of elastic medium due to couple-stress effect. All these variations are shown in Fig. 4. In Fig. 5, the amplitude ratio $|z_5|$ of refracted Sv-wave in elastic medium has been magnified by multiplying its original value by 10. The comparison of amplitude ratio $|z_5|$ of refracted Sv-wave in couple-stress medium with amplitude ratio $|z_5|$ reveals that both approaches to nearby same value at $\theta^0 = 90^0$, but has a sharp difference at the initial value. The variation of the amplitude ratios of $|z_3|$ and $|z_6|$ of reflected ss-wav have been depicted in Figs.6 and 7 respectively.

Sub Case (b) : Incident SV-wave

The amplitude ratio $|z_1|$ of reflected P-wave in couple stress elastic medium decreases sharply up to $\theta_0 = 10^0$, decreases monotonically for $10^0 \le \theta_0 \le 33^0$, oscillates for the range $33^0 \le \theta_0 \le 72^0$ and again decreases as θ_0 increases further. The value of the amplitude ratio $|z_1|$ of reflected P-wave in elastic medium has been magnified by multiplying its original value by 10 and these variations are shown in Fig. 8.

Fit 9 depicts the variations of the amplitude ratio $|z_2|$ of the reflected Sv-wave in couple stress and elastic medium and it is observed that there is sharp difference between the maximum values of these amplitude ratios.

The amplitude ratio $|z_3|$ of refracted P-wave in elastic medium has its max. value at $\theta_0 = 2^0$ and decreases first sharply and then gradually. The value of the amplitude ratio $|z_4|$ of refracted P-wave in couple stress medium has been magnified by multiplying its original value by 10 and these variations are shown in Fig. 10.

Fig. 11 shows the variations of the amplitude ratio $|z_5|$ of reflected Sv-wave in couple stress elastic medium and the amplitude ratio $|z_5|$ of refracted Sv-wave in elastic medium.

The variations of the amplitude ratios $|z_3|$ and $|z_6|$ of the reflected SS-wave and refracted SS-wave respectively in couple stress elastic medium have been shown in Figs. 12 and 13 respectively.

CASE II. Couple-Stress elastic half-space

Subcase (a) : Incident P-wave

When we compare the amplitude ratio $|z_1|$ of reflected P-wave in couple-stress elastic half space with the amplitude ratio $|z_1|$ of reflected P-wave in elastic half space, we observe that due to the effect of couple stress value of the amplitude $|z_1|$ of elastic half space increases, though both of these approaches same value at $\theta_0 = 90^0$ and these variations are shown in Fig. 14.

The amplitude ratios $|z_2|$ of reflected Sv-wave in coupled-stress elastic half space and the amplitude ratio $|z_2|$ of reflected Sv-wave in elastic half space have almost same value except the max. value and around it and these variations are shown in Fig. 15.

The variation of amplitude ratio $|z_3|$ of SS-wave in coupled half space has been shown in Fig. 18.

Subcase (b) : Incident Sv-wave

The comparison of the amplitude ratio $|z_1|$ of reflected P-wave in coupled stress elastic half space with the amplitude ratio $|z_1|$ of reflected P-wave in elastic half space has been shown in fig. 16. The value of amplitude ratio $|z_1|$ of elastic half space has been magnified by multiplying its original value by 10, which is almost similar to the max. value of $|z_1|$ of elastic half space.

The amplitude ratio $|z_2|$ of reflected Sv-wave in coupled half space has its max. value at $\theta_0 = 49^0$, which is very large in comparison to the max. value of the amplitude ratio $|z_2|$ of reflected Sv-wave in elastic half-space and these variations are shown in Fig. 17.

The variation of amplitude ratio $|z_3|$ of SS-wave in coupled half space has been shown in Fig 19.

CONCLUSION

The analytical expressions for reflection and transmission coefficients of various reflected waves are derived. The variations of the reflection and transmission coefficients of various reflected waves have been depicted graphically. Some particular cases have been deduced. It may be concluded that, the effect of couple stress plays an important role in a reflection and refraction phenomena. The model adopted in this paper is one of the realistic forms of the earth model and it may be of interest for experimental seismologists.

LIST OF SYMBOLS

λ, μ = Lame's constants

η = The bending twist modulus,

ρ = The density

$\in_{ijk}$ = Alternating Tensor

δ , ω_j = Kronecker Delta , rotational vector respectively

η' = The bending of twisting modulus

l = The dimension of length and it carries the effect of couple-stress

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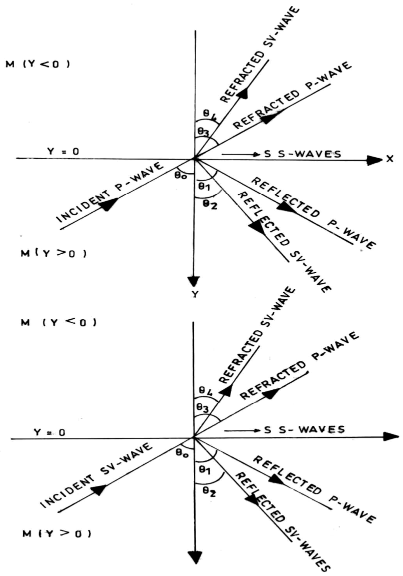

Fig.1 Geometry of the problem.

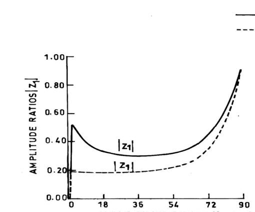

FIG. 2

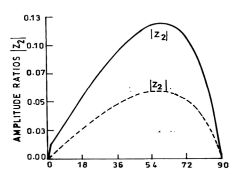

FIG. 3

Figs 2-3 Variations of amplitude ratios with incident angle of P-wave at couple stress/couple-stress elastic interface.

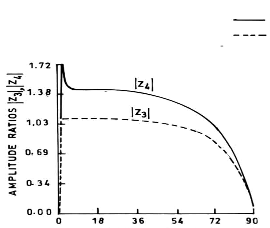

FIG. 4

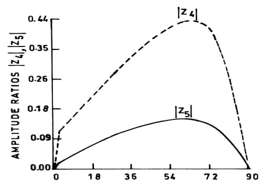

FIG. 5

Figs 4-5 Variations of amplitude ratios with incident angle of P-wave at couple stress/ couple-stress elastic interface

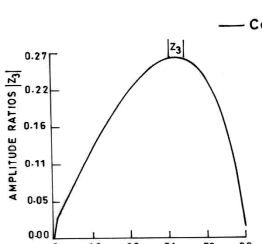

FIG. 6

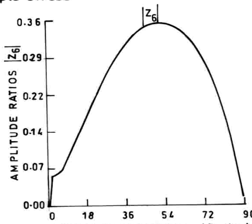

FIG. 7

Figs 6-7 Variations of amplitude ratios with incident angle of P-wave at couple stress/couple- stress elastic interface

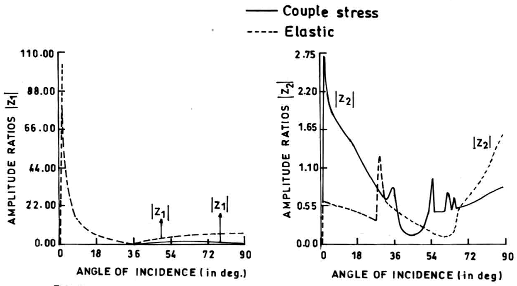

FIG. 8 FIG.9

Figs 8-9 Variations of amplitude ratios with incident angle of Sv-wave at couple stress/couple stress-elastic interface

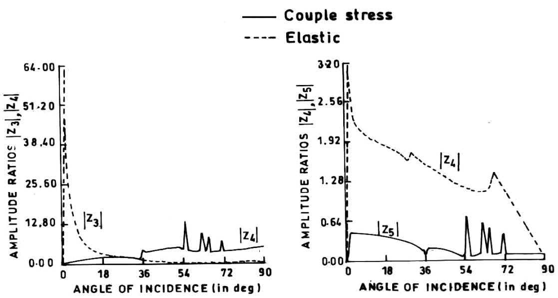

FIG.10 FIG.11

Fig. 10-11 Variations of amplitude ratios with incident angle of Sv-wave at couple stress/ couple-stress elastic interface

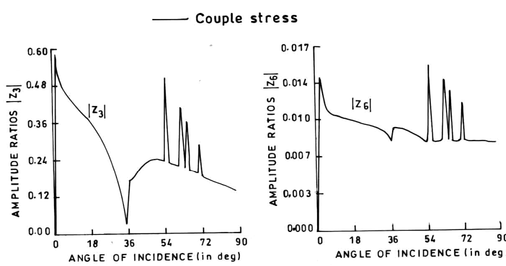

FIG 12 FIG.13

Figs 12-13 Variations of amplitude ratios with incident angle of Sv-wave at couple stress/couple-stress elastic interface

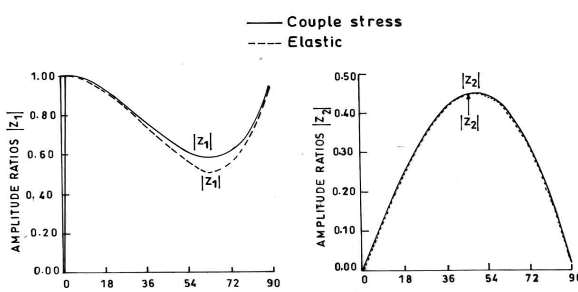

FIG.14 FIG.15

Figs 14-15 Variations of amplitude ratios with incident angle of P-wave at couple stress elastic half-space.

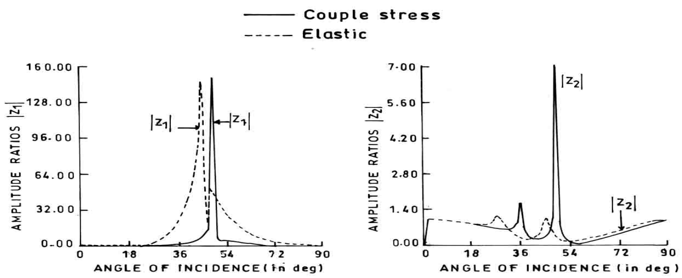

FIG.16

FIG.17

Figs 16-17 Variations of amplitude ratios with incident angle of Sv-wave at couple stress elastic half-space.

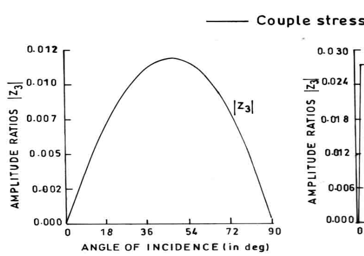

Fig.18 Variations of amplitude ratios with incident angle of P-wave at couple stress elastic half-space.

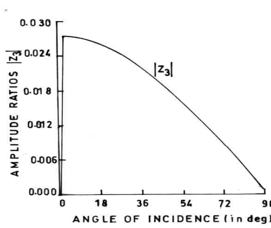

Fig.19 Variations of amplitude ratios with incident angle of Sv-wave at couple stress elastic half-space.