A STUDY ON SOME OPERATIONS OF FUZZY SOFT SETS

Manoj Borah[1], Tridiv Jyoti Neog[2], Dusmanta Kumar Sut[3]
[1]Deptt. of Mathematics, Jorhat Institute of Science & Technology, Jorhat, Assam, India
[2]Deptt. of Mathematics, D.K. High School, Jorhat,
[3]Deptt. of Mathematics, N.N.Saikia College, Titabor, Assam, India

ABSTRACT
The purpose of this paper is to study some operations and results available in the literature of fuzzy soft sets. Instead of taking the notion of complement of a fuzzy soft set put forward by Maji, throughout our work, we have taken the notion of complement of a fuzzy soft set put forward by Neog and Sut.

Keywords – Fuzzy Set, Soft Set, Fuzzy Soft Set.

1. INTRODUCTION
In many complicated problems arising in the fields of engineering, social science, economics, medical science etc involving uncertainties, classical methods are found to be inadequate in recent times. Molodtsov [**2**] pointed out that the important existing theories viz. Probability Theory, Fuzzy Set Theory, Intuitionistic Fuzzy Set Theory, Rough Set Theory etc. which can be considered as mathematical tools for dealing with uncertainties, have their own difficulties. He further pointed out that the reason for these difficulties is, possibly, the inadequacy of the parameterization tool of the theory. In 1999 he initiated the novel concept of Soft Set as a new mathematical tool for dealing with uncertainties. Soft Set Theory, initiated by Molodtsov [**2**], is free of the difficulties present in these theories. In 2011, Neog and Sut [**9**] put forward a new notion of complement of a soft set and accordingly some important results have been studied in their work.
In recent times, researches have contributed a lot towards fuzzification of Soft Set Theory. Maji et al. [**6**] introduced the concept of Fuzzy Soft Set and some properties regarding fuzzy soft union, intersection, complement of a fuzzy soft set, De Morgan Law etc. These results were further revised and improved by Ahmad and Kharal [**1**]. Recently, Neog and Sut [**8**] have studied the notions of fuzzy soft union, fuzzy soft intersection, complement of a fuzzy soft set and several other properties of fuzzy soft sets along with examples and proofs of certain results.
In this paper, we have studied some operations and results available in the literature of fuzzy soft sets. Instead of taking the notion of complement of a fuzzy soft set put forward by Maji et al. [**6**] , throughout our work, we have taken the notion of complement of a fuzzy soft set put forward by Neog and Sut [**7**].

2. PRELIMINARIES
In this section, we first recall the basic definitions related to soft sets and fuzzy soft sets which would be used in the sequel.

2.1. Soft Set [2]
A pair (*F*, *E*) is called a soft set (over *U*) if and only if *F* is a mapping of *E* into the set of all subsets of the set *U*.
In other words, the soft set is a parameterized family of subsets of the set *U*. Every set $F(\varepsilon), \varepsilon \in E$, from this family may be considered as the set of ε - elements of the soft set (*F*, *E*), or as the set of ε - approximate elements of the soft set.

2.2. Soft Null Set [5]
A soft set (*F*, *A*) over *U* is said to be null soft set denoted by $\tilde{\varphi}$ if $\forall \varepsilon \in A, F(\varepsilon) = \varphi$ (Null set)

2.3. Soft Absolute Set [5]
A soft set (*F*, *A*) over *U* is said to be absolute soft set denoted by $\tilde{A}$ if $\forall \varepsilon \in A, F(\varepsilon) = U$.

2.4. Soft Subset [5]
For two soft sets (*F*, *A*) and (*G*, *B*) over the universe *U*, we say that (*F*, *A*) is a soft subset of (*G*, *B*), if
(*i*) $A \subset B$,
(*ii*) $\forall \varepsilon \in A, F(\varepsilon)$ and $G(\varepsilon)$ are identical approximations
and is written as $(F, A) \tilde{\subseteq} (G, B)$.

Pei and Miao [**4**] modified this definition of soft subset in the following way –

2.5. Soft Subset Redefined [4]
For two soft sets (*F*, *A*) and (*G*, *B*) over the universe *U*, we say that (*F*, *A*) is a soft subset of (*G*, *B*), if
(*i*) $A \subseteq B$,
(*ii*) $\forall \varepsilon \in A, F(\varepsilon) \subseteq G(\varepsilon)$ and is written as $(F, A) \tilde{\subseteq} (G, B)$.
(*F*, *A*) is said to be soft superset of (*G*, *B*) if (*G*, *B*) is a soft subset of (*F*, *A*) and we write $(F, A) \tilde{\supseteq} (G, B)$.

2.6. Union of Soft Sets [5]
Union of two soft sets (*F*, *A*) and (*G*, *B*) over a common universe *U*, is the soft set (*H*, *C*), where $C = A \cup B$ and $\forall \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & \text{if } \varepsilon \in A - B \\ G(\varepsilon), & \text{if } \varepsilon \in B - A \\ F(\varepsilon) \cup G(\varepsilon), & \text{if } \varepsilon \in A \cap B \end{cases}$$

and is written as $(F, A) \tilde{\cup} (G, B) = (H, C)$.

2.7. Intersection of Soft Sets [5]

Intersection of two soft sets (*F*, *A*) and (*G*, *B*) over a common universe *U*, is the soft set (*H*, *C*), where $C = A \cap B$ and $\forall \varepsilon \in C$, $H(\varepsilon) = F(\varepsilon)$ or $G(\varepsilon)$ (as both are same set) and is written as $(F, A)\tilde{\cap}(G, B) = (H, C)$.

Pei and Miao [**4**] pointed out that generally $F(\varepsilon)$ or $G(\varepsilon)$ may not be identical. Moreover in order to avoid the degenerate case, Ahmad and Kharal [**1**] proposed that $A \cap B$ must be non-empty and thus revised the above definition as follows.

2.8. Intersection of Soft Sets Redefined [1]

Let (*F*, *A*) and (*G*, *B*) be two soft sets over a common universe *U* with $A \cap B \neq \phi$. Then Intersection of two soft sets (*F*, *A*) and (*G*, *B*) is a soft set (*H*, *C*) where $C = A \cap B$ and $\forall \varepsilon \in C$, $H(\varepsilon) = F(\varepsilon) \cap G(\varepsilon)$.
We write $(F, A)\tilde{\cap}(G, B) = (H, C)$.

2.9. AND Operation of Soft Sets [5]

If (*F*, *A*) and (*G*, *B*) be two soft sets, then "(*F*, *A*) AND (*G*, *B*)" is a soft set denoted by $(F, A) \wedge (G, B)$ and is defined by $(F, A) \wedge (G, B) = (H, A \times B)$, where $H(\alpha, \beta) = F(\alpha) \cap G(\beta), \forall \alpha \in A$ and $\forall \beta \in B$, where $\cap$ is the operation intersection of two sets.

2.10. OR Operation of Soft Sets [5]

If (*F*, *A*) and (*G*, *B*) be two soft sets, then "(*F*, *A*) OR (*G*, *B*)" is a soft set denoted by $(F, A) \vee (G, B)$ and is defined by $(F, A) \vee (G, B) = (K, A \times B)$, where $K(\alpha, \beta) = F(\alpha) \cup G(\beta), \forall \alpha \in A$ and $\forall \beta \in B$, where $\cup$ is the operation union of two sets.

2.11. Complement of a Soft Set [9]

The complement of a soft set (*F*, *A*) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$, where $F^c: A \rightarrow P(U)$ is a mapping given by $F^c(\varepsilon) = [F(\varepsilon)]^c$ for all $\varepsilon \in A$.

2.12. Fuzzy Soft Set [6]

A pair (*F*, *A*) is called a fuzzy soft set over *U* where $F : A \rightarrow \tilde{P}(U)$ is a mapping from *A* into $\tilde{P}(U)$.

2.13. Fuzzy Soft Class [1]

Let *U* be a universe and *E* a set of attributes. Then the pair (*U*, *E*) denotes the collection of all fuzzy soft sets on *U* with attributes from *E* and is called a fuzzy soft class.

2.14. Fuzzy Soft Null Set [6]

A soft set (*F*, *A*) over *U* is said to be null fuzzy soft set denoted by φ if $\forall \varepsilon \in A, F(\varepsilon)$ is the null fuzzy set $\bar{0}$ of *U* where $\bar{0}(x) = 0 \forall x \in U$.

2.15. Fuzzy Soft Absolute Set [6]

A soft set (*F*, *A*) over *U* is said to be absolute fuzzy soft set denoted by $\tilde{A}$ if $\forall \varepsilon \in A, F(\varepsilon)$ is the absolute fuzzy set $\bar{1}$ of *U* where $\bar{1}(x) = 1 \forall x \in U$

2.16. Fuzzy Soft Subset [6]

For two fuzzy soft sets (*F*, *A*) and (*G*, *B*) in a fuzzy soft class (*U*, *E*), we say that (*F*, *A*) is a fuzzy soft subset of (*G*, *B*), if
(*i*) $A \subseteq B$
(*ii*) For all $\varepsilon \in A, F(\varepsilon) \subseteq G(\varepsilon)$ and is written as
$(F, A) \tilde{\subseteq} (G, B)$.

2.17. Union of Fuzzy Soft Sets [6]

Union of two fuzzy soft sets (*F*, *A*) and (*G*, *B*) in a soft class (*U*, *E*) is a fuzzy soft set (*H*, *C*) where $C = A \cup B$ and $\forall \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & \text{if } \varepsilon \in A - B \\ G(\varepsilon), & \text{if } \varepsilon \in B - A \\ F(\varepsilon) \cup G(\varepsilon), & \text{if } \varepsilon \in A \cap B \end{cases}$$

And is written as $(F, A)\tilde{\cup}(G, B) = (H, C)$.

2.18. Intersection of Fuzzy Soft Sets [6]

Intersection of two fuzzy soft sets (*F*, *A*) and (*G*, *B*) in a soft class (*U*, *E*) is a fuzzy soft set (*H*, *C*) where $C = A \cap B$ and $\forall \varepsilon \in C$, $H(\varepsilon) = F(\varepsilon)$ or $G(\varepsilon)$ (as both are same fuzzy set) and is written as $(F, A)\tilde{\cap}(G, B) = (H, C)$.

Ahmad and Kharal [**1**] pointed out that generally $F(\varepsilon)$ or $G(\varepsilon)$ may not be identical. Moreover in order to avoid the degenerate case, he proposed that $A \cap B$ must be non-empty and thus revised the above definition as follows -

2.19. Intersection of Fuzzy Soft Sets Redefined [1]

Let (*F*, *A*) and (*G*, *B*) be two fuzzy soft sets in a soft class (*U*, *E*) with $A \cap B \neq \phi$.Then Intersection of two fuzzy soft sets (*F*, *A*) and (*G*, *B*) in a soft class (*U*, *E*) is a fuzzy soft set (*H*,*C*) where $C = A \cap B$ and $\forall \varepsilon \in C$,
$H(\varepsilon) = F(\varepsilon) \cap G(\varepsilon)$. We write $(F, A)\tilde{\cap}(G, B) = (H, C)$.

2.20. Complement of a Fuzzy Soft Set [7]

The complement of a fuzzy soft set (*F*, *A*) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$ where $F^c : A \rightarrow \tilde{P}(U)$ is a mapping given by $F^c(\alpha) = [F(\alpha)]^c$, $\forall \alpha \in A$.

2.21. AND Operation of Fuzzy Soft Sets [6]

If (*F*, *A*) and (*G*, *B*) be two fuzzy soft sets, then "(*F*, *A*) AND (*G*, *B*)" is a fuzzy soft set denoted by $(F, A) \wedge (G, B)$ and is defined by $(F, A) \wedge (G, B) = (H, A \times B)$, where $H(\alpha, \beta) = F(\alpha) \cap G(\beta), \forall \alpha \in A$ and $\forall \beta \in B$, where $\cap$ is the operation intersection of two fuzzy sets.

2.22. OR Operation of Soft Sets [6]

If (*F*, *A*) and (*G*, *B*) be two fuzzy soft sets, then "(*F*, *A*) OR (*G*, *B*)" is a fuzzy soft set denoted by $(F, A) \vee (G, B)$ and is defined by $(F, A) \vee (G, B) = (K, A \times B)$, where $K(\alpha, \beta) = F(\alpha) \cup G(\beta), \forall \alpha \in A$ and $\forall \beta \in B$, where $\cup$ is the operation union of two fuzzy sets.

3. A STUDY ON THE OPERATIONS IN FUZZY SOFT SETS

In this section, we shall endeavour to study the basic operations and results available in the literature of fuzzy soft sets. Ahmad and Kharal [**1**] has pointed out that the null (resp., absolute) fuzzy soft set as defined by Maji et al. [**6**], is not unique in a fuzzy soft class (*U*, *E*), rather it depends upon *A* ⊆ *E*. As such whenever we would refer to a fuzzy soft null set or fuzzy soft absolute set, we would refer to the set *A* ⊆ *E* of parameters under consideration. In case of soft sets, this has already been pointed out by Ge and Yang in [**10**] and accordingly they have studied some basic results regarding soft sets. Here, we put forward some results of fuzzy soft sets in our way. We are using the notation (φ, A) to represent the fuzzy soft null set with respect to the set of parameters *A* and the notation (U, A) to represent the fuzzy soft absolute set with respect to the set of parameters *A*.

3.1. Proposition

1.$(\varphi, A)^c = (U, A)$

Proof. Let $(\varphi, A) = (F, A)$

Then $\forall \varepsilon \in A$,

$$F(\varepsilon) = \{(x, \mu_{F(\varepsilon)}(x)) : x \in U\} = \{(x, 0) : x \in U\}$$

$$(\varphi, A)^c = (F, A)^c = (F^c, A), \text{ where}$$

$$\begin{aligned}\forall \varepsilon \in A,\ F^c(\varepsilon) &= (F(\varepsilon))^c \\ &= \{(x, \mu_{F(\varepsilon)}(x)) : x \in U\}^c \\ &= \{(x, 1 - \mu_{F(\varepsilon)}(x)) : x \in U\} \\ &= \{(x, 1 - 0) : x \in U\} \\ &= \{(x, 1) : x \in U\} \\ &= U\end{aligned}$$

Thus $(\varphi, A)^c = (U, A)$

2.$(U, A)^c = (\varphi, A)$

Proof. Let $(U, A) = (F, A)$

Then $\forall \varepsilon \in A$,

$$F(\varepsilon) = \{(x, \mu_{F(\varepsilon)}(x)) : x \in U\} = \{(x, 1) : x \in U\}$$

$$(U, A)^c = (F, A)^c = (F^c, A), \text{ where}$$

$$\begin{aligned}\forall \varepsilon \in A,\ F^c(\varepsilon) &= (F(\varepsilon))^c \\ &= \{(x, \mu_{F(\varepsilon)}(x)) : x \in U\}^c \\ &= \{(x, 1 - \mu_{F(\varepsilon)}(x)) : x \in U\} \\ &= \{(x, 1 - 1) : x \in U\} \\ &= \{(x, 0) : x \in U\} \\ &= \varphi\end{aligned}$$

Thus $(U, A)^c = (\varphi, A)$

3.$(F, A) \tilde{\cup} (\varphi, A) = (F, A)$

Proof. We have

$$\begin{aligned}(F, A) &= \{(\varepsilon, (x, \mu_{F(\varepsilon)}(x))) : x \in U\}\ \forall \varepsilon \in A \\ (\varphi, A) &= \{(\varepsilon, (x, 0)) : x \in U\}\ \forall \varepsilon \in A \\ (F, A) \tilde{\cup} (\varphi, A) &= \{(\varepsilon, (x, \max(\mu_{F(\varepsilon)}(x), 0))) : x \in U\}\ \forall \varepsilon \in A \\ &= \{(\varepsilon, (x, \mu_{F(\varepsilon)}(x))) : x \in U\}\ \forall \varepsilon \in A \\ &= (F, A)\end{aligned}$$

Thus $(F, A) \tilde{\cup} (\varphi, A) = (F, A)$

4.$(F, A) \tilde{\cup} (U, A) = (U, A)$

Proof. We have

$$\begin{aligned}(F, A) &= \{(\varepsilon, (x, \mu_{F(\varepsilon)}(x))) : x \in U\}\ \forall \varepsilon \in A \\ (U, A) &= \{(\varepsilon, (x, 1)) : x \in U\}\ \forall \varepsilon \in A \\ (F, A) \tilde{\cup} (U, A) &= \{(\varepsilon, (x, \max(\mu_{F(\varepsilon)}(x), 1))) : x \in U\}\ \forall \varepsilon \in A \\ &= \{(\varepsilon, (x, 1)) : x \in U\}\ \forall \varepsilon \in A \\ &= (U, A)\end{aligned}$$

Thus $(F, A) \tilde{\cup} (U, A) = (U, A)$

5.$(F, A) \tilde{\cap} (\varphi, A) = (\varphi, A)$

Proof. We have

$$\begin{aligned}(F, A) &= \{(\varepsilon, (x, \mu_{F(\varepsilon)}(x))) : x \in U\}\ \forall \varepsilon \in A \\ (\varphi, A) &= \{(\varepsilon, (x, 0)) : x \in U\}\ \forall \varepsilon \in A \\ (F, A) \tilde{\cap} (\varphi, A) &= \{(\varepsilon, (x, \min(\mu_{F(\varepsilon)}(x), 0))) : x \in U\}\ \forall \varepsilon \in A \\ &= \{(\varepsilon, (x, 0)) : x \in U\}\ \forall \varepsilon \in A \\ &= (\varphi, A)\end{aligned}$$

Thus $(F, A) \tilde{\cap} (\varphi, A) = (\varphi, A)$

6.$(F, A) \tilde{\cap} (U, A) = (F, A)$

Proof. We have

$$\begin{aligned}(F, A) &= \{(\varepsilon, (x, \mu_{F(\varepsilon)}(x))) : x \in U\}\ \forall \varepsilon \in A \\ (U, A) &= \{(\varepsilon, (x, 1)) : x \in U\}\ \forall \varepsilon \in A\end{aligned}$$

$$\begin{aligned}(F,A)\tilde{\cup}(U,A) &= \{(\varepsilon,(x,\min(\mu_{F(\varepsilon)}(x),1))): x\in U\} \quad \forall\varepsilon\in A\\ &= \{(\varepsilon,(x,\mu_{F(\varepsilon)}(x))): x\in U\} \quad \forall\varepsilon\in A\\ &= (F,A)\end{aligned}$$

Thus $(F,A)\tilde{\cap}(U,A)=(F,A)$

7. $(F,A)\tilde{\cup}(\varphi,B)=(F,A)$ if and only if $B\subseteq A$

Proof.
We have for (F,A)

$$F(\varepsilon) = \{(x,\mu_{F(\varepsilon)}(x)): x\in U\} \quad \forall\varepsilon\in A$$

Also, let $(\varphi,B)=(G,B)$, Then

$$G(\varepsilon) = \{(x,0): x\in U\} \ \forall\varepsilon\in B$$

Let $(F,A)\tilde{\cup}(\varphi,B)=(F,A)\tilde{\cup}(G,B)=(H,C)$, where $C=A\cup B$ and $\forall\varepsilon\in C$,

$H(\varepsilon)$

$$=\begin{cases}\{(x,\mu_{F(\varepsilon)}(x)): x\in U\}, \text{if } \varepsilon\in A-B\\ \{(x,\mu_{G(\varepsilon)}(x)): x\in U\}, \text{if } \varepsilon\in B-A\\ \{(x,\max(\mu_{F(\varepsilon)}(x),\mu_{G(\varepsilon)}(x))): x\in U\}, \text{if } \varepsilon\in A\cap B\end{cases}$$

$$=\begin{cases}\{(x,\mu_{F(\varepsilon)}(x)): x\in U\}, \text{if } \varepsilon\in A-B\\ \{(x,0): x\in U\}, \text{if } \varepsilon\in B-A\\ \{(x,\max(\mu_{F(\varepsilon)}(x),0)): x\in U\}, \text{if } \varepsilon\in A\cap B\end{cases}$$

$$=\begin{cases}\{(x,\mu_{F(\varepsilon)}(x)): x\in U\}, \text{if } \varepsilon\in A-B\\ \{(x,0): x\in U\}, \text{if } \varepsilon\in B-A\\ \{(x,\mu_{F(\varepsilon)}(x)): x\in U\}, \text{if } \varepsilon\in A\cap B\end{cases}$$

Let $B\subseteq A$

Then

$$H(\varepsilon) = \begin{cases}\{(x,\mu_{F(\varepsilon)}(x)): x\in U\}, \text{if } \varepsilon\in A-B\\ \{(x,\mu_{F(\varepsilon)}(x)): x\in U\}, \text{if } \varepsilon\in A\cap B\end{cases}$$

$$= F(\varepsilon) \ \forall\varepsilon\in A$$

Conversely, let $(F,A)\tilde{\cup}(\varphi,B)=(F,A)$

Then $A=A\cup B \Rightarrow B\subseteq A$

8. $(F,A)\tilde{\cup}(U,B)=(U,B)$ if and only if $A\subseteq B$

Proof.
We have for (F,A)

$$F(\varepsilon) = \{(x,\mu_{F(\varepsilon)}(x)): x\in U\} \quad \forall\varepsilon\in A$$

Also, let $(U,B)=(G,B)$, Then

$$G(\varepsilon) = \{(x,1): x\in U\} \ \forall\varepsilon\in B$$

Let $(F,A)\tilde{\cup}(U,B)=(F,A)\tilde{\cup}(G,B)=(H,C)$, where $C=A\cup B$ and $\forall\varepsilon\in C$,

$H(\varepsilon)$

$$=\begin{cases}\{(x,\mu_{F(\varepsilon)}(x)): x\in U\}, \text{if } \varepsilon\in A-B\\ \{(x,\mu_{G(\varepsilon)}(x)): x\in U\}, \text{if } \varepsilon\in B-A\\ \{(x,\max(\mu_{F(\varepsilon)}(x),\mu_{G(\varepsilon)}(x))): x\in U\}, \text{if } \varepsilon\in A\cap B\end{cases}$$

$$=\begin{cases}\{(x,\mu_{F(\varepsilon)}(x)): x\in U\}, \text{if } \varepsilon\in A-B\\ \{(x,1): x\in U\}, \text{if } \varepsilon\in B-A\\ \{(x,\max(\mu_{F(\varepsilon)}(x),1)): x\in U\}, \text{if } \varepsilon\in A\cap B\end{cases}$$

$$=\begin{cases}\{(x,\mu_{F(\varepsilon)}(x)): x\in U\}, \text{if } \varepsilon\in A-B\\ \{(x,1): x\in U\}, \text{if } \varepsilon\in B-A\\ \{(x,1): x\in U\}, \text{if } \varepsilon\in A\cap B\end{cases}$$

Let $A\subseteq B$

Then

$$H(\varepsilon) = \begin{cases}\{(x,1): x\in U\}, \text{if } \varepsilon\in B-A\\ \{(x,1): x\in U\}, \text{if } \varepsilon\in A\cap B\end{cases}$$

$$= G(\varepsilon) \ \forall\varepsilon\in B$$

Conversely, let $(F,A)\tilde{\cup}(U,B)=(U,B)$

Then $B=A\cup B \Rightarrow A\subseteq B$

9. $(F,A)\tilde{\cap}(\varphi,B)=(\varphi,A\cap B)$

Proof.
We have for (F,A)

$$F(\varepsilon) = \{(x,\mu_{F(\varepsilon)}(x)): x\in U\} \quad \forall\varepsilon\in A$$

Also, let $(\varphi,B)=(G,B)$, Then

$$G(\varepsilon) = \{(x,0): x\in U\} \ \forall\varepsilon\in B$$

Let $(F,A)\tilde{\cap}(\varphi,B)=(F,A)\tilde{\cap}(G,B)=(H,C)$, where $C=A\cap B$ and $\forall\varepsilon\in C$,

$$\begin{aligned}H(\varepsilon) &= \{(x,\min(\mu_{F(\varepsilon)}(x),\mu_{G(\varepsilon)}(x))): x\in U\}\\ &= \{(x,\min(\mu_{F(\varepsilon)}(x),0)): x\in U\}\\ &= \{(x,0): x\in U\}\end{aligned}$$

Thus $(F,A)\tilde{\cup}(\varphi,B)=(\varphi,A\cap B)$

10. $(F,A)\tilde{\cap}(U,B)=(F,A\cap B)$

Proof.
We have for (F,A)

$$F(\varepsilon) = \{(x,\mu_{F(\varepsilon)}(x)): x\in U\} \quad \forall\varepsilon\in A$$

Also, let $(U,B)=(G,B)$, Then

$$G(\varepsilon) = \{(x,1): x\in U\} \ \forall\varepsilon\in B$$

Let $(F,A)\tilde{\cap}(U,B)=(F,A)\tilde{\cap}(G,B)=(H,C)$, where $C=A\cap B$ and $\forall\varepsilon\in C$,

$$\begin{aligned}H(\varepsilon) &= \{(x,\min(\mu_{F(\varepsilon)}(x),\mu_{G(\varepsilon)}(x))): x\in U\}\\ &= \{(x,\min(\mu_{F(\varepsilon)}(x),1)): x\in U\}\end{aligned}$$

$= \{(x, \mu_{F(\varepsilon)}(x)) : x \in U\}$

Thus $(F, A) \tilde{\cap} (U, B) = (F, A \cap B)$

It is well known that De Morgan Laws inter-relate union and intersection via complements. Maji et al [**6**] gave the following proposition-

3.2. Proposition

1. $((F, A) \tilde{\cup} (G, B))^c = (F, A)^c \tilde{\cup} (G, B)^c$
2. $((F, A) \tilde{\cap} (G, B))^c = (F, A)^c \tilde{\cap} (G, B)^c$

Ahmad and Kharal [**1**] proved by counter examples that these propositions are not valid. However the following inclusions are due to Ahmad and Kharal [**1**]. They proved these results with the definition of complement initiated by Maji et al. [**6**]. Below we are giving the proof in our way.

3.3. Proposition

For fuzzy soft sets (*F*, *A*) and (*G*, *B*) over the same universe *U*, we have the following -

1. $((F, A) \tilde{\cup} (G, B))^c \tilde{\subseteq} (F, A)^c \tilde{\cup} (G, B)^c$
2. $(F, A)^c \tilde{\cap} (G, B)^c \tilde{\subseteq} ((F, A) \tilde{\cap} (G, B))^c$

Proof.

1. Let $(F, A) \tilde{\cup} (G, B) = (H, C)$, where $C = A \cup B$ and $\forall \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), \text{if } \varepsilon \in A\text{-}B \\ G(\varepsilon), \text{if } \varepsilon \in B\text{-}A \\ F(\varepsilon) \cup G(\varepsilon), \text{if } \varepsilon \in A \cap B \end{cases}$$

$$= \begin{cases} \{x, \mu_{F(\varepsilon)}(x)\}, \text{if } \varepsilon \in A\text{-}B \\ \{x, \mu_{G(\varepsilon)}(x)\}, \text{if } \varepsilon \in B\text{-}A \\ \{x, \max(\mu_{F(\varepsilon)}, \mu_{G(\varepsilon)})\}, \text{if } \varepsilon \in A \cap B \end{cases}$$

Thus

$((F, A) \tilde{\cup} (G, B))^c = (H, C)^c = (H^c, C)$, where $C = A \cup B$ and $\forall \varepsilon \in C$,

$$H^c(\varepsilon) = (H(\varepsilon))^c$$

$$= \begin{cases} (F(\varepsilon))^c, \text{if } \varepsilon \in A\text{-}B \\ (G(\varepsilon))^c, \text{if } \varepsilon \in B\text{-}A \\ (F(\varepsilon) \cup G(\varepsilon))^c, \text{if } \varepsilon \in A \cap B \end{cases}$$

$$= \begin{cases} \{x, 1 - \mu_{F(\varepsilon)}(x)\}, \text{if } \varepsilon \in A\text{-}B \\ \{x, 1 - \mu_{G(\varepsilon)}(x)\}, \text{if } \varepsilon \in B\text{-}A \\ \{x, 1 - \max(\mu_{F(\varepsilon)}, \mu_{G(\varepsilon)})\}, \text{if } \varepsilon \in A \cap B \end{cases}$$

$$= \begin{cases} \{x, 1 - \mu_{F(\varepsilon)}(x)\}, \text{if } \varepsilon \in A\text{-}B \\ \{x, 1 - \mu_{G(\varepsilon)}(x)\}, \text{if } \varepsilon \in B\text{-}A \\ \{x, \min(1 - \mu_{F(\varepsilon)}(x), 1 - \mu_{G(\varepsilon)}(x))\}, \text{if } \varepsilon \in A \cap B \end{cases}$$

Again,

$(F, A)^c \tilde{\cup} (G, B)^c = (F^c, A) \tilde{\cup} (G^c, B) = (I, J)$, say

Where $J = A \cup B$ and $\forall \varepsilon \in J$,

$$I(\varepsilon) = \begin{cases} F^c(\varepsilon), \text{if } \varepsilon \in A\text{-}B \\ G^c(\varepsilon), \text{if } \varepsilon \in B\text{-}A \\ F^c(\varepsilon) \cup G^c(\varepsilon), \text{if } \varepsilon \in A \cap B \end{cases}$$

$$= \begin{cases} \{x, \mu_{F^c(\varepsilon)}(x)\}, \text{if } \varepsilon \in A\text{-}B \\ \{x, \mu_{G^c(\varepsilon)}(x)\}, \text{if } \varepsilon \in B\text{-}A \\ \{x, \max(\mu_{F^c(\varepsilon)}(x), \mu_{G^c(\varepsilon)}(x))\}, \text{if } \varepsilon \in A \cap B \end{cases}$$

$$= \begin{cases} \{x, 1 - \mu_{F(\varepsilon)}(x)\}, \text{if } \varepsilon \in A\text{-}B \\ \{x, 1 - \mu_{G(\varepsilon)}(x)\}, \text{if } \varepsilon \in B\text{-}A \\ \{x, \max(1 - \mu_{F(\varepsilon)}, 1 - \mu_{G(\varepsilon)})\}, \text{if } \varepsilon \in A \cap B \end{cases}$$

We see that *C* = *J* and $\forall \varepsilon \in C, H^c(\varepsilon) \subseteq I(\varepsilon)$

Thus $((F, A) \tilde{\cup} (G, B))^c \tilde{\subseteq} (F, A)^c \tilde{\cup} (G, B)^c$

2. Let $(F, A) \tilde{\cap} (G, B) = (H, C)$,

Where $C = A \cap B$ and $\forall \varepsilon \in C$,

$$H(\varepsilon) = F(\varepsilon) \cap G(\varepsilon)$$
$$= \{x, \min(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x))\}$$

Thus $((F, A) \tilde{\cap} (G, B))^c = (H, C)^c = (H^c, C)$,

Where $C = A \cap B$ and $\forall \varepsilon \in C$,

$$H^c(\varepsilon) = (F(\varepsilon) \cap G(\varepsilon))^c$$
$$= \{x, 1 - \min(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x))\}$$
$$= \{x, \max(1 - \mu_{F(\varepsilon)}(x), 1 - \mu_{G(\varepsilon)}(x))\}$$

Again, $(F, A)^c \tilde{\cap} (G, B)^c = (F^c, A) \tilde{\cap} (G^c, B) = (I, J)$, say

Where $J = A \cap B$ and $\forall \varepsilon \in J$,

$$I(\varepsilon) = F^c(\varepsilon) \cap G^c(\varepsilon)$$
$$= \{x, \min(\mu_{F^c(\varepsilon)}(x), \mu_{G^c(\varepsilon)}(x))\}$$
$$= \{x, \min(1 - \mu_{F(\varepsilon)}(x), 1 - \mu_{G(\varepsilon)}(x))\}$$

We see that *C* = *J* and $\forall \varepsilon \in C, I(\varepsilon) \subseteq H^c(\varepsilon)$

Thus $(F, A)^c \tilde{\cap} (G, B)^c \tilde{\subseteq} ((F, A) \tilde{\cap} (G, B))^c$

3.4. Proposition (De Morgan Inclusions)

For fuzzy soft sets (*F*, *A*) and (*G*, *B*) over the same universe *U*, we have the following -

1. $(F, A)^c \tilde{\cap} (G, B)^c \tilde{\subseteq} ((F, A) \tilde{\cup} (G, B))^c$
2. $((F, A) \tilde{\cap} (G, B))^c \tilde{\subseteq} (F, A)^c \tilde{\cup} (G, B)^c$

Proof

1. Let $(F, A) \tilde{\cup} (G, B) = (H, C)$, where $C = A \cup B$ and $\forall \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), \text{if } \varepsilon \in A\text{-}B \\ G(\varepsilon), \text{if } \varepsilon \in B\text{-}A \\ F(\varepsilon) \cup G(\varepsilon), \text{if } \varepsilon \in A \cap B \end{cases}$$

$$= \begin{cases} \{x, \mu_{F(\varepsilon)}(x)\}, \text{if } \varepsilon \in A\text{-}B \\ \{x, \mu_{G(\varepsilon)}(x)\}, \text{if } \varepsilon \in B\text{-}A \\ \{x, \max(\mu_{F(\varepsilon)}, \mu_{G(\varepsilon)})\}, \text{if } \varepsilon \in A \cap B \end{cases}$$

Thus

$((F,A)\tilde{\cup}(G,B))^c = (H,C)^c = (H^c, C)$, where $C = A \cup B$ and $\forall \varepsilon \in C$,

$$H^c(\varepsilon) = (H(\varepsilon))^c$$

$$= \begin{cases} (F(\varepsilon))^c, \text{if } \varepsilon \in A\text{-}B \\ (G(\varepsilon))^c, \text{if } \varepsilon \in B\text{-}A \\ (F(\varepsilon) \cup G(\varepsilon))^c, \text{if } \varepsilon \in A \cap B \end{cases}$$

$$= \begin{cases} \{x, 1-\mu_{F(\varepsilon)}(x)\}, \text{if } \varepsilon \in A\text{-}B \\ \{x, 1-\mu_{G(\varepsilon)}(x)\}, \text{if } \varepsilon \in B\text{-}A \\ \{x, 1-\max(\mu_{F(\varepsilon)}, \mu_{G(\varepsilon)})\}, \text{if } \varepsilon \in A \cap B \end{cases}$$

$$= \begin{cases} \{x, 1-\mu_{F(\varepsilon)}(x)\}, \text{if } \varepsilon \in A\text{-}B \\ \{x, 1-\mu_{G(\varepsilon)}(x)\}, \text{if } \varepsilon \in B\text{-}A \\ \{x, \min(1-\mu_{F(\varepsilon)}(x), 1-\mu_{G(\varepsilon)}(x))\}, \text{if } \varepsilon \in A \cap B \end{cases}$$

Again, $(F,A)^c \tilde{\cap} (G,B)^c = (F^c, A)\tilde{\cap}(G^c, B) = (I, J)$, say

Where $J = A \cap B$ and $\forall \varepsilon \in J$,

$$I(\varepsilon) = F^c(\varepsilon) \cap G^c(\varepsilon)$$

$$= \{x, \min(\mu_{F^c(\varepsilon)}(x), \mu_{G^c(\varepsilon)}(x))\}$$

$$= \{x, \min(1-\mu_{F(\varepsilon)}(x), 1-\mu_{G(\varepsilon)}(x))\}$$

We see that $J \subseteq C$ and $\forall \varepsilon \in J, I(\varepsilon) = H^c(\varepsilon)$

Thus $(F,A)^c \tilde{\cap} (G,B)^c \tilde{\subseteq} ((F,A)\tilde{\cup}(G,B))^c$

2. Let $(F,A)\tilde{\cap}(G,B) = (H,C)$, Where $C = A \cap B$ and $\forall \varepsilon \in C$,

$$H(\varepsilon) = F(\varepsilon) \cap G(\varepsilon)$$

$$= \{x, \min(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x))\}$$

Thus $((F,A)\tilde{\cap}(G,B))^c = (H,C)^c = (H^c, C)$,

Where $C = A \cap B$ and $\forall \varepsilon \in C$,

$$H^c(\varepsilon) = (F(\varepsilon) \cap G(\varepsilon))^c$$

$$= \{x, 1-\min(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x))\}$$

$$= \{x, \max(1-\mu_{F(\varepsilon)}(x), 1-\mu_{G(\varepsilon)}(x))\}$$

Again,

$(F,A)^c \tilde{\cup} (G,B)^c = (F^c, A)\tilde{\cup}(G^c, B) = (I, J)$, say

Where $J = A \cup B$ and $\forall \varepsilon \in J$,

$$I(\varepsilon) = \begin{cases} F^c(\varepsilon), \text{if } \varepsilon \in A\text{-}B \\ G^c(\varepsilon), \text{if } \varepsilon \in B\text{-}A \\ F^c(\varepsilon) \cup G^c(\varepsilon), \text{if } \varepsilon \in A \cap B \end{cases}$$

$$= \begin{cases} \{x, \mu_{F^c(\varepsilon)}(x)\}, \text{if } \varepsilon \in A\text{-}B \\ \{x, \mu_{G^c(\varepsilon)}(x)\}, \text{if } \varepsilon \in B\text{-}A \\ \{x, \max(\mu_{F^c(\varepsilon)}(x), \mu_{G^c(\varepsilon)}(x))\}, \text{if } \varepsilon \in A \cap B \end{cases}$$

$$= \begin{cases} \{x, 1-\mu_{F(\varepsilon)}(x)\}, \text{if } \varepsilon \in A\text{-}B \\ \{x, 1-\mu_{G(\varepsilon)}(x)\}, \text{if } \varepsilon \in B\text{-}A \\ \{x, \max(1-\mu_{F(\varepsilon)}, 1-\mu_{G(\varepsilon)})\}, \text{if } \varepsilon \in A \cap B \end{cases}$$

We see that $C \subseteq J$ and $\forall \varepsilon \in C, H^c(\varepsilon) = I(\varepsilon)$

Thus $((F,A)\tilde{\cap}(G,B))^c \tilde{\subseteq} (F,A)^c \tilde{\cup} (G,B)^c$

De Morgan Laws are valid for fuzzy soft sets with the same set of parameter. Thus for fuzzy soft sets, we have the following De Morgan Laws:

3.5. Proposition (De Morgan Laws)

For fuzzy soft sets (F, A) and (G, A) over the same universe U, we have the following -

1. $((F,A)\tilde{\cup}(G,A))^c = (F,A)^c \tilde{\cap} (G,A)^c$
2. $((F,A)\tilde{\cap}(G,A))^c = (F,A)^c \tilde{\cup} (G,A)^c$

Proof.

1. Let $(F,A)\tilde{\cup}(G,A) = (H,A)$, where $\forall \varepsilon \in A$,

$$H(\varepsilon) = F(\varepsilon) \cup G(\varepsilon)$$

$$= \{x, \max(\mu_{F(\varepsilon)}, \mu_{G(\varepsilon)})\}$$

Thus

$((F,A)\tilde{\cup}(G,A))^c = (H,A)^c = (H^c, A)$, where $\forall \varepsilon \in A$,

$$H^c(\varepsilon) = (H(\varepsilon))^c$$

$$= (F(\varepsilon) \cup G(\varepsilon))^c$$

$$= \{x, 1-\max(\mu_{F(\varepsilon)}, \mu_{G(\varepsilon)})\}$$

$$= \{x, \min(1-\mu_{F(\varepsilon)}(x), 1-\mu_{G(\varepsilon)}(x))\}$$

Again, $(F,A)^c \tilde{\cap} (G,A)^c = (F^c, A)\tilde{\cap}(G^c, A) = (I, A)$, say

Where $\forall \varepsilon \in A$,

$$I(\varepsilon) = F^c(\varepsilon) \cap G^c(\varepsilon)$$

$$= \{x, \min(\mu_{F^c(\varepsilon)}(x), \mu_{G^c(\varepsilon)}(x))\}$$

$$= \{x, \min(1-\mu_{F(\varepsilon)}(x), 1-\mu_{G(\varepsilon)}(x))\}$$

Thus $((F,A)\tilde{\cup}(G,A))^c = (F,A)^c \tilde{\cap} (G,A)^c$

2. Let $(F,A)\tilde{\cap}(G,A) = (H,A)$, where $\forall \varepsilon \in A$,

$$H(\varepsilon) = F(\varepsilon) \cap G(\varepsilon)$$

$$= \{x, \min(\mu_{F(\varepsilon)}, \mu_{G(\varepsilon)})\}$$

Thus

$((F,A)\tilde{\cap}(G,A))^c = (H,A)^c = (H^c, A)$, where $\forall \varepsilon \in A$,

$$H^c(\varepsilon) = (H(\varepsilon))^c$$

$$= (F(\varepsilon) \cap G(\varepsilon))^c$$

$$= \{x, 1-\min(\mu_{F(\varepsilon)}, \mu_{G(\varepsilon)})\}$$

$= \{x, \max(1-\mu_{F(\varepsilon)}(x), 1-\mu_{G(\varepsilon)}(x))\}$

Again, $(F,A)^c \,\tilde{\cup}\, (G,A)^c = (F^c, A) \,\tilde{\cup}\, (G^c, A) = (I, A)$, say

Where $\forall \varepsilon \in A$,

$$\begin{aligned} I(\varepsilon) &= F^c(\varepsilon) \cup G^c(\varepsilon) \\ &= \{x, \max(\mu_{F^c(\varepsilon)}(x), \mu_{G^c(\varepsilon)}(x))\} \\ &= \{x, \max(1-\mu_{F(\varepsilon)}(x), 1-\mu_{G(\varepsilon)}(x))\} \end{aligned}$$

Thus $((F,A) \,\tilde{\cup}\, (G,A))^c = (F,A)^c \,\tilde{\cap}\, (G,A)^c$

Maji et al [**6**] proved the following De Morgan Types of results for fuzzy soft sets (*F*, *A*) and (*G*, *B*) over the same universe *U*. We can verify that these De Morgan types of results are valid in our way also.

3.6. Proposition

For fuzzy soft sets (*F*, *A*) and (*G*, *B*) over the same universe *U*, we have the following -

1. $((F,A) \wedge (G,B))^c = (F,A)^c \vee (G,B)^c$
2. $((F,A) \vee (G,B))^c = (F,A)^c \wedge (G,B)^c$

Proof.

1. Let $(F,A) \wedge (G,B) = (H, A \times B)$,

Where $H(\alpha,\beta) = F(\alpha) \cap G(\beta),\ \forall \alpha \in A$ and $\forall \beta \in B$,

where $\cap$ is the operation intersection of two fuzzy sets. Thus

$$\begin{aligned} H(\alpha,\beta) &= F(\alpha) \cap G(\beta) \\ &= \{x, \min(\mu_{F(\alpha)}(x), \mu_{G(\beta)}(x))\} \end{aligned}$$

Thus

$$\begin{aligned} ((F,A) \wedge (G,B))^c &= (H, A \times B)^c \\ &= (H^c, A \times B), \text{ where } \forall (\alpha,\beta) \in A \times B, \\ H^c(\alpha,\beta) &= (H(\alpha,\beta))^c \\ &= \{x, 1-\min(\mu_{F(\alpha)}(x), \mu_{G(\beta)}(x))\} \\ &= \{x, \max(1-\mu_{F(\alpha)}(x), 1-\mu_{G(\beta)}(x))\} \end{aligned}$$

Let $(F,A)^c \vee (G,B)^c = (F^c, A) \vee (G^c, B) = (O, A \times B)$,

Where $O(\alpha,\beta) = F^c(\alpha) \cup G^c(\beta),\ \forall \alpha \in A$ and $\forall \beta \in B$,

where $\cup$ is the operation union of two fuzzy sets.

$$\begin{aligned} &= \{x, \max(\mu_{F^c(\alpha)}(x), \mu_{G^c(\beta)}(x))\} \\ &= \{x, \max(1-\mu_{F(\alpha)}(x), 1-\mu_{G(\beta)}(x))\} \end{aligned}$$

It follows that $((F,A) \wedge (G,B))^c = (F,A)^c \vee (G,B)^c$

2. Let $(F,A) \vee (G,B) = (H, A \times B)$,

Where $H(\alpha,\beta) = F(\alpha) \cup G(\beta),\ \forall \alpha \in A$ and $\forall \beta \in B$,

where $\cup$ is the operation union of two sets.

Thus

$$\begin{aligned} H(\alpha,\beta) &= F(\alpha) \cup G(\beta) \\ &= \{x, \max(\mu_{F(\alpha)}(x), \mu_{G(\beta)}(x))\} \end{aligned}$$

Thus

$$\begin{aligned} ((F,A) \vee (G,B))^c &= (H, A \times B)^c \\ &= (H^c, A \times B), \text{ where } \forall (\alpha,\beta) \in A \times B, \\ H^c(\alpha,\beta) &= (H(\alpha,\beta))^c \\ &= \{x, 1-\max(\mu_{F(\alpha)}(x), \mu_{G(\beta)}(x))\} \\ &= \{x, \min(1-\mu_{F(\alpha)}(x), 1-\mu_{G(\beta)}(x))\} \end{aligned}$$

Let $(F,A)^c \wedge (G,B)^c = (F^c, A) \wedge (G^c, B) = (O, A \times B)$,

Where $O(\alpha,\beta) = F^c(\alpha) \cap G^c(\beta),\ \forall \alpha \in A$ and $\forall \beta \in B$,

where $\cap$ is the operation union of two fuzzy sets.

$$\begin{aligned} &= \{x, \min(\mu_{F^c(\alpha)}(x), \mu_{G^c(\beta)}(x))\} \\ &= \{x, \min(1-\mu_{F(\alpha)}(x), 1-\mu_{G(\beta)}(x))\} \end{aligned}$$

It follows that $((F,A) \vee (G,B))^c = (F,A)^c \wedge (G,B)^c$

4. CONCLUSION

We have made an investigation on existing basic notions and results on fuzzy soft sets. Some new results have been stated in our work. Future work in this regard would be required to study whether the notions put forward in this paper yield a fruitful result

5. REFERENCES

[1] B. Ahmad and A. Kharal, "*On Fuzzy Soft Sets*", Advances in Fuzzy Systems, Volume 2009.

[2] D. A. Molodtsov, "*Soft Set Theory - First Result*", Computers and Mathematics with Applications, Vol. 37, pp. 19-31, 1999

[3] D. Chen, E.C.C. Tsang, D.S. Yeung, and X.Wang , "*The parameterization reduction of soft sets and its applications*," *Computers & Mathematics with Applications*, vol. 49, no.5-6, pp. 757–763, 2005.

[4] D. Pei and D. Miao, "*From soft sets to information systems*," in *Proceedings of the IEEE International Conference on Granular Computing*, vol. 2, pp. 617–621, 2005.

[5] P. K. Maji and A.R. Roy, "*Soft Set Theory*", Computers and Mathematics with Applications 45 (2003) 555 – 562

[6] P. K. Maji, R. Biswas and A.R. Roy, "*Fuzzy Soft Sets*", Journal of Fuzzy Mathematics, Vol 9 , no.3,pp.-589-602,2001

[7] T. J. Neog, D. K. Sut, "*On Fuzzy Soft Complement and Related Properties*" , Accepted for publication in International Journal of Energy, Information and communications (IJEIC).

[8] T. J. Neog, D. K. Sut, "*On Union and Intersection of Fuzzy Soft Sets*", Int.J. Comp. Tech. Appl., Vol 2 (5), 1160-1176

[9] T. J. Neog and D. K. Sut, " *A New Approach To The Theory of Soft Sets*" , International Journal of Computer Applications, Vol 32, No 2,October 2011, pp 1-6

[10] Xun Ge and Songlin Yang, " *Investigations on some operations of soft sets*", World Academy of Science, Engineering and Technology, 75, 2011, pp. 1113-1116