Performance Analysis of MIMO Systems using Orthogonal Space Time Coding over Rayleigh Fading Channel

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Abstract: Emerging demands for high data rate services and high spectral efficiency are the key driving forces for the continued technology evolution in wireless communications. MIMO technology has attracted attention in wireless communications, because it offers significant increases in data throughput and link range without additional bandwidth or transmit power. It achieves this by higher spectral efficiency (more bits per second per hertz of bandwidth) and link reliability or diversity (reduced fading). Orthogonal space-time block codes (STBC's) have received considerable attention in recent open-loop multiple-input-multiple-output (MIMO) wireless communication because they allow low decoding complexity and guarantee full diversity. This paper presents a detailed study of space-time block coding (STBC) schemes including orthogonal STBC for 3×4 antennas and high-coding rate STBC. Finally, STBC 3×4 techniques is implemented in MATLAB and analyzed the performance according to their bit-error rates using BPSK, QPSK.

Keywords: Diversity, Multiple Input and Multiple Output (MIMO), Orthogonal space- time coding (OSTBC), Channel State information (CSI), Pair-wise Error Probability.

I.INTRODUCTION

MIMO wireless systems have captured the attention of international standard organizations. The use of MIMO has been proposed multiple times for use in the high-speed packet data mode of third generation cellular systems (3G) as well as the fourth generation cellular systems (4G). MIMO has also influenced wireless local area networks (WLANs) as the IEEE 802.11n standard exploits the use of MIMO systems to acquire high throughputs. MIMO systems employing space-time coding strategies to support greatly enhanced performance. **S**pace-Time coding [5], [4], uses the advantage of transmitter diversity, is an effective technique to improve the performance of wireless communication systems. In space-time coding, different signals are simultaneously transmitted from different transmit antennas. The signal which is received is the superposition of the different transmitted signals, and the detection process needs estimates of the channel parameters [3]. All these designs were based on the assumption that channel state information is perfectly known at the receiver, but unknown at the transmitter.

The work presented in this paper is motivated by the observation that for the special case of STBC 3×4(3transmitter and 4 receiver) and high code-rate STBC, it is possible to obtain exact closed-form expression for the pair-wise error probability. An exact PEP expression would serve as an attractive alternative to previously derived bounds for evaluating performance [6]. Our expressions are derived from the PDF of the phase of the received signal. Simulated PEP results of STBC 3×4 using BPSK and QPSK are presented.

II.System model: MIMO

When a transmitter and a receiver, with an appropriate channel coding and decoding scheme, are equipped with multiple antennas, the presence of multipath fading can be improved over a Raleigh fading channel.

Space-time-coded MIMO systems with N_T transmit antennas and N_R receive antennas is showed in the figure. In the space-time coded MIMO systems, bit stream is mapped into symbol stream$\{\tilde{x}_i\}_{i=1}^{N}$. As depicted in Figure a symbol stream of size N is space-time-encoded into $\{\tilde{x}_i\}_{i=1}^{N}$ t=1, 2, 3….T, where i is the antenna index and t is the symbol time index. Note that the number of symbols in a space-time codeword is N_T .T (i.e., N = $N_T \times$ T). In other words $\{\tilde{x}_i\}_{i=1}^{N}$, t=1, 2, 3…T, forms a space-time codeword. As N symbols are transmitted by a codeword over T symbol times, the symbol rate of the space-time-coded system example shown in the figure is given as

$$R = \frac{N}{T}(\text{Symbols / Channel use}) \qquad (1)$$

At the receiver side, the symbol stream $\{\tilde{x}_i\}_{i=1}^{N}$is estimated by using the receive signals $\left\{y_j^{(t)}\right\}_{i=1}^{N_R}$, t= 1, 2,….. T. Let h_{ij}^{t} denotes the Rayleigh-distributed channel gain from the i^{th} transmit antenna to the j^{th} receive antenna over the t^{th} symbol period (i = 1; 2; . . . ; N_T),(j = 1; 2; . . . ; N_R), and t = 1; 2; . . . ; T). If we assume that the channel gains do not change during T symbol periods, the symbol time index $\{h_{ji}^{t}\}$can be omitted. Furthermore, as long as the transmit antennas and receive antennas are spaced sufficiently apart, $N_R \times N_T$ fading gains can be assumed to be statistically independent [3]

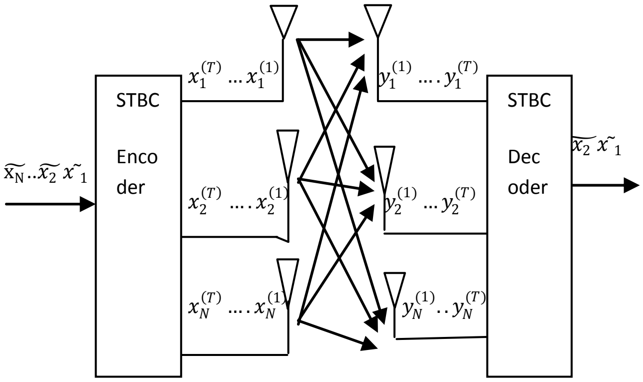

Figure.1 Block diagram of Space-time coded MIMO systems

. If x_i^t is the transmitted signal from the i^{th} transmit antenna during t^{th} symbol period, the received signal at the j^{th} receive antenna during t^{th} symbol period is

$$y_j^{(t)} = \sqrt{\frac{E_X}{N_O N_T}}\left[h_{j1}^{(t)} h_{j2}^{(t)} \ldots\ldots h_{j3}^{(t)}\right]\begin{bmatrix} x_1^{(t)} \\ x_2^{(t)} \\ \vdots \\ x_{N_T}^{(t)} \end{bmatrix} + z_j^{(t)} \qquad (2)$$

Where Z_j^t is the noise process at the j^{th} receive antenna during t^{th} symbol period, which is modeled as the ZMCSCG noise of unit variance, and E_X is the average energy of each transmitted signal. Meanwhile, the total transmitted power is constrained as

$$\sum_{i=1}^{N_T} E\left\{\left|x_i^{(t)}\right|^2\right\} = N_T, \quad t=1,2,\ldots\ldots\ldots T \qquad (3)$$

Variance is assumed to be 0.5 for real and imaginary parts of h_{ij}.

Considering the relationship i Equation (2) for N_R receive antennas, while assuming quasi-static channel gains (i.e.$h_{ji}^t = h_{ji}, t = 1,2,\ldots.T$), the system equation is given as

$$\begin{bmatrix} y_1^{(1)} & y_1^{(2)}\ldots & y_1^{(T)} \\ \vdots & \vdots & \vdots \\ y_{N_R}^{(1)} & y_{N_R}^{(2)}\ldots & y_{N_R}^{(T)} \end{bmatrix} = \sqrt{\frac{E_x}{N_O N_T}}\begin{bmatrix} h_{11} & h_{12}\ldots & h_{1N_T} \\ \vdots & \vdots & \vdots \\ h_{N_R 1} & h_{N_R 2}\ldots & h_{N_R N_T} \end{bmatrix}\begin{bmatrix} x_1^{(1)} & x_1^{(2)}\ldots & x_1^{(T)} \\ \vdots & \vdots & \vdots \\ x_{N_T}^{(1)} & x_{N_T\ldots}^{(2)} & x_{N_T}^{(T)} \end{bmatrix} + \begin{bmatrix} Z_1^{(1)} & Z_1^{(2)}\ldots & Z_1^{(3)} \\ \vdots & \vdots & \vdots \\ Z_{N_R}^{(1)} & Z_{N_R}^{(2)} & Z_{N_R}^{(T)} \end{bmatrix} \qquad (4)$$

III.Orthogonal Space Time Block Codes

In higher order STBC in-order to facilitate computationally-efficient ML detection at the receiver, the following property is required:

$$XX^H = c(|x_i^1|^2 + |x_i^2|^2 + \cdots..|x_i^T|^2)I_{N_T} = c\|x_i\|^2 I_{N_T} \qquad (5)$$

Consider transmitting antennas N_T=3 transmitting complex space time block codes in 8 time slots with coding rate of 1/2, while satisfying a full rank condition

$$X_{3,complex}^{low\ rate} = \begin{bmatrix} x_1 & -x_2 & -x_3 & -x_4 & x_1^* & -x_2^* & -x_3^* & -x_4^* \\ x_2 & x_1 & x_4 & -x_3 & x_2^* & x_1^* & x_4^* & -x_3^* \\ x_3 & -x_4 & x_1 & x_2 & x_3^* & -x_4^* & x_1^* & x_2^* \end{bmatrix} \qquad (6)$$

Space-time block codes can be used for various numbers of receive antennas. However, only a single receive antenna is assumed. We express the received signals from a single receive antenna as

$$[y_1 y_2 y_3 y_4 y_5 y_6 y_7 y_8] = \sqrt{\frac{E_x}{3N_O}}[h_1 h_2 h_3]\begin{bmatrix} x_1 & -x_2 & -x_3 & -x_4 & x_1^* & -x_2^* & -x_3^* & -x_4^* \\ x_2 & x_1 & x_4 & -x_3 & x_2^* & x_1^* & x_4^* & -x_3^* \\ x_3 & -x_4 & x_1 & x_2 & x_3^* & -x_4^* & x_1^* & x_2^* \end{bmatrix} + [Z_1 Z_2 Z_3 Z_4 Z_5 Z_6 Z_7 Z_8] \qquad (7)$$

The above input-output relation can be also expressed as

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \end{bmatrix} = \sqrt{\frac{E_x}{3N_O}}\begin{bmatrix} h_1 & h_2 & h_3 & 0 \\ h_2 & -h_1 & 0 & -h_3 \\ h_3 & 0 & -h_1 & h_2 \\ 0 & h_3 & -h_2 & -h_1 \\ h_1^* & h_2^* & h_3 & 0 \\ h_2^* & -h_1^* & 0 & -h_3^* \\ h_3^* & 0 & -h_1^* & h_2^* \\ 0 & h_3 & -h_2^* & h_1^* \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \\ Z_6 \\ Z_7 \\ Z_8 \end{bmatrix} \qquad (8)$$

Again, using the orthogonality of the above effective channel matrix, the received signal is modified as

$$y_{eff} = H_{eff}^H y_{eff} = 2\sqrt{\frac{E_x}{3N_o}}\sum_{j=1}^{3}|h_j|^2 I_4 X_{eff} + Z_{eff} \qquad (9)$$

Using the above result, the ML signal detection is performed as

$$x_{i,ML} = Q\left(\frac{y_{eff,i}}{2\sqrt{\frac{E_X}{3N_o}}\sum_1^3|h_j|^2}\right) \quad i = 1,2,3,4 \qquad (10)$$

If a decoding complexity at the receiver is compromised, however, higher coding rates can be achieved by the following codes:

$$X_{3,complex}^{high\ rate} = \begin{bmatrix} x_1 & -x_2^* & \frac{x_3^*}{\sqrt{2}} & \frac{x_3^*}{\sqrt{2}} \\ x_2 & x_1^* & \frac{x_3^*}{\sqrt{2}} & \frac{-x_3^*}{\sqrt{2}} \\ \frac{x_3}{\sqrt{2}} & \frac{x_3}{\sqrt{2}} & \frac{(-x_1-x_1^*+x_2-x_2^*)}{2} & \frac{(x_2+x_2^*+x_1-x_1^*)}{2} \end{bmatrix} \quad (11)$$

Coding rate is R = ¾

Decoding of higher coding rates can be achieved by the following equations

$$[y_1 y_2 y_3 y_4] = \sqrt{\frac{E_x}{3N_O}}[h_1 h_2 h_3]\begin{bmatrix} x_1 & -x_2^* & \frac{x_3^*}{\sqrt{2}} & \frac{x_3^*}{\sqrt{2}} \\ x_2 & x_1^* & \frac{x_3^*}{\sqrt{2}} & \frac{-x_3^*}{\sqrt{2}} & 0 \\ \frac{x_3}{\sqrt{2}} & \frac{x_3}{\sqrt{2}} & \frac{(-x_1-x_1^*+x_2-x_2^*)}{2} & \frac{(x_2+x_2^*+x_1-x_1^*)}{2} \end{bmatrix} + [Z_1 Z_2 Z_3 Z_4] \quad (12)$$

Using the equation (12), the ML signal detection is performed as

$$x_{i,ML} = Q\left(\frac{y_{eff,i}}{2\sqrt{\frac{E_X}{3N_o}}\sum_1^3 |h_j|^2}\right) \quad i = 1,2,3,4 \quad (13)$$

Effective channel construction for $X_{3,complex}^{high\ rate}$ is rather more complex than the previous coding.

IV. Pair-wise error Probability

Pair-wise error probability is defined as probability of transmitting C^1 and detecting it as C^2, when there is no other code-words. It is represented as

$$P(C^1 \longrightarrow C^2).$$

Conditional probability is written as, refer to [6].

$$P(C^1 \longrightarrow C^2/H) = Q\left(\sqrt{\frac{\gamma}{2}Tr[H^H(C^2-C^1)^H.(C^2-C^1)H]}\right) \quad (14)$$

According to the Orthogonality conditional PEP is written as

$$P(C^1 \longrightarrow C^2/H) = Q\left(\sqrt{\frac{\gamma}{2}k\sum_{k=1}^{k}|S_k^2 - S_k^1|^2 Tr[H^H.H]}\right) = Q\left(\sqrt{\frac{\gamma}{2}k\sum_{k=1}^{k}|S_k^2 - S_k^1|^2\sum_{n=1}^{N}\sum_{m=1}^{M}|\alpha_{n,m}|^2}\right) \quad (15)$$

Euclidian distance between T_X and detected symbol is given by

$$d_E = \sqrt{\sum_{k=1}^{k}|S_k^2 - S_k^1|^2}$$

$$P(C^1 \longrightarrow C^2/H) = Q\left(\sqrt{\frac{\gamma}{2}kd_E^2\sum_{n=1}^{N}\sum_{m=1}^{M}|\alpha_{n,m}|^2}\right) \quad (16)$$

To calculate PEP, one needs to integrate above equation weighted by density of path gains

$$P(C^1 \longrightarrow C^2/H) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} exp\left(\frac{-k\gamma d_E^2\sum_{n=1}^{N}\sum_{m=1}^{M}|\alpha_{n,m}|^2}{4sin\Phi^2}\right)d\boldsymbol{\Phi}$$

$$= \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\prod_{m=1}^{M}\prod_{n=1}^{N}.exp\left(\frac{-k\gamma d_{E|\alpha_{n,m}|^2}^2}{4sin\Phi^2}\right)d\boldsymbol{\Phi} \quad (17)$$

Let the path gains are independent from each other. The integral over the distribution of the path gains is same as product of MN equal integrals

i.e. $\prod_{m=1}^{M}\prod_{n=1}^{N}.=\int_0^{\infty}.$

$$P(C^1 \longrightarrow C^2/H) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\left[\int_0^{\infty}.exp\left(\frac{-k\gamma d_E^2 x}{4sin\Phi^2}\right)\delta_{\chi(x)}dx\right]^{MN}d\Phi$$

Where $f_\chi(x) = e^{-x}$,x> 0 *is the pdf of* $|\alpha_{n,m}|^2$. Moment Generating Function of exponential distribution for μ< 1 *is given by*

$$M_\chi(\mu) = E[e^{\mu\chi}] = \int_0^{\infty}.e^{\mu\chi}f_\chi(x)dx = \int_0^{\infty}.e^{\mu\chi}e^{-x}dx = \frac{1}{1-\mu} \quad (18)$$

Since $\mu = \frac{-k\gamma d_E^2 x}{4sin\Phi^2}$

$$P(C^1 \longrightarrow C^2) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\frac{1}{1+\frac{K\gamma d_E^2}{4\sin\Phi^2}}$$

$$= \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\left[\frac{\sin\Phi^2}{\sin\Phi^2 + k\frac{\gamma}{4}d_E^2}\right]^{MN}d\Phi \quad (19)$$

The generalized expression of PEP for STBC is given by

$$P(C^1 \longrightarrow C^2) = \frac{1}{2}\left\{1 - \sqrt{\frac{a}{1+a}}\sum_{i=0}^{MN-1}\binom{2i}{i}\left[\frac{1}{4(1+a)}\right]^i\right\}$$

Where a = $k\frac{\gamma}{4}d_E^2$ (20)

N=Number of Transmitters

M=Number of receivers

V.Simulation Results

In this work, MATLAB is used to test the BER performance of the Rayleigh fading channel model for STBC with transmitters ($N_T = 3$) and receivers (N_R=4) for different code rates. Results are shown below. From the results it is observed that in the case of BPSK the BER decrease as SNR increases rather than in the case of QPSK.

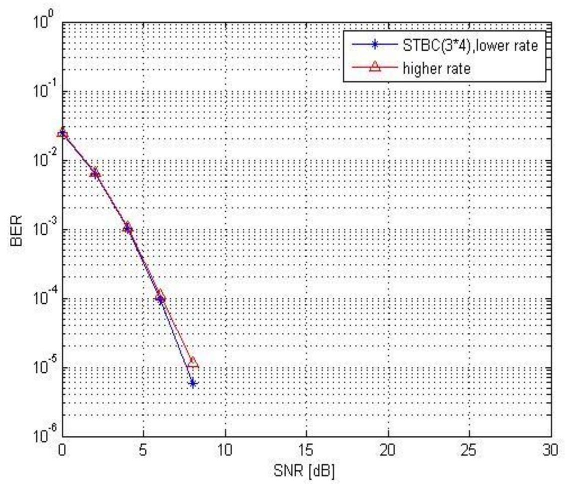

Figure 2.BER versus SNR of STBC using BPSK

By using the complex space time block code shown in the equation (6) with low code rate and equation (11) with high code rate, it is also observed that nearly same probability of error can be achieved.

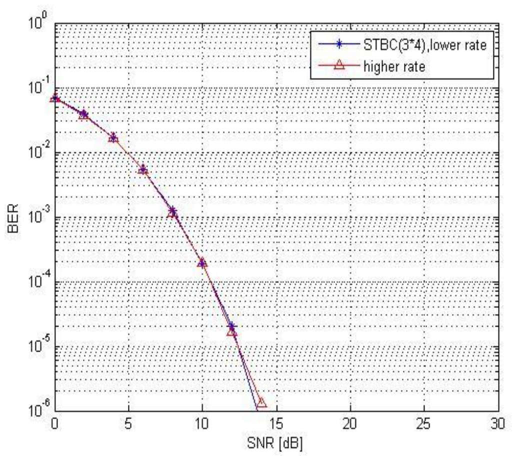

Figure 3.BER versus SNR of STBC using QPSK

VI. Conclusion

This paper provides need and advantages of MIMO systems. A basic introduction to Space-Time Coding was provided by presenting STBC system model. We then discussed block codes schemes for the cases of 3transmit antennas and 4 receive antennas .High data rate code Scheme also discussed .The encoding and decoding algorithm for each were presented. Generalized pair-wise error probability (PEP) for the STBC was presented. Finally from the simulation results we conclude that data rate can be increased by using high rate code in STBC. It is also seen that same BER can be achieved in STBC by using high rate codes as that of using low rate codes if decoding complexity is compromised at the receiver.

REFERENCES

[1] H. Sampath, S. Talwar, J. Tellado, V. Erceg, and A. Paulraj, "A fourth-generation MIMO-OFDM broadband wireless system: design, performance, and field trial results," *IEEE Communications Magazine, vol. 40, no. 9, pp. 143–149, 2002.*

[2] A. Molisch, Wireless Communications. Wiley-IEEE Press, 2005

[3] V. Tarokh, H. Jafarkhani, and A. Calderbank, "Space-time block coding for wireless communications: performance results," *IEEE Journal on selected areas in communications, vol. 17, no. 3, pp. 451–460, 1999*

[4] G. Tsoulos, MIMO system technology for wireless communications. *CRC Press,2006.*

[5] B. Vucetic and J. Yuan, *Space-Time Coding*, England, Wiley, 2003.

[6] V. Tarokh, H. Jafarkhami, and A.R. Calderbank, "Space-time block codes from orthogonal designs", *IEEE Transactions on Information Theory, Vol. 45, No. 5, pp. 1456-1467, July 1999.*

[7] G. G. Raleigh and J. M. Cioffi, "Spatio-Temporal Coding for Wireless communication", *IEEE Transaction on Communication,* Vol. 46, No. 3, pp. 357-366, March 1998.

[8] M.K. Simon, "Evaluation of average bit error probability for space-time coding based on a simpler exact evaluation of pair-wise error probability," *International Journal on Communications and Networks*, vol. 3, no. 3, pp. 257–264, Sept. 2001.

[9] M. Uysal and C. N. Georghiades, "Error performance analysis of spacetime codes over Rayleigh fading channels," *J. Commun. Networks, vol. 2, no. 4, pp. 351–355, Dec. 2000*.

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