

Singularity Spectrum Analysis of Electrical Demand Time Series

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ABSTRACT

This study estimates systematically the dynamics of electrical power demand functions for two countries India and China using singularity spectrum analysis. In order to demonstrate the capability of fractal approach on electrical power demand, we choose Wavelet Transform Modulus maxima (WTMM) technology and results imply that this method is powerful in studying the singularities of time series signals. We show that the WTMM, the so called multi-resolution analysis, can determine the singularity spectrum of multi-fractal measures from the scaling behavior of the partition function, and account for the multi-fractal nature of the time series obtained from the chaotic phenomena. Applying the WTMM we will be able to verify that the electrical demand is a persistent series and this method can be used to compare the quality of the process in two different places. The correlation dimensions found tells us that the series obtained for China is long-range correlated than India with long term memory. This lays that China power generating system is better suited to satisfy oscillations in the demand.

Keywords – Wavelet Transform Modulus Maxima, Singularity spectrum, Lipschitz holder exponent, Developing countries, Electricity demand, Multi-fractal analysis.

I. INTRODUCTION

With an electricity demand of 322 KWh per capita per year in 2003, India's electricity demand has been growing at an average of 8.8% per year from 1986 to 2003 while the peak demand increased on average by 6.3% per annum from 540 MW to 1516 MW. The electricity demand growth, GDP growth and electricity price variation for India for the period 1986-2003, illustrates the relatively moderate Indian GDP Growth (averaging 3.5% per annum from 1986-2003), but despite this, India's per capita electricity consumption is still somewhat lower than of its neighbors China and Pakistan although both countries have experienced much lower per capita income levels. Although these economies are not directly compatible with Indian economy, they are the closest geographical neighbors to India with some cultural and trade links. Although these economies are not directly compatible with Indian economy, they are the closest geographical neighbors to India with some cultural and trade links. India consumed 439 kg of oil equivalent per person of primary energy in 2003 compared to 1090 in China, 7835 in US and the world

average of 1688. India's energy use efficiency for generating GDP in purchasing power parity terms is better than the world average. China, US, and Germany. However, it is 7% to 23% higher than Denmark, UK, and Japan and Brazil. If we look at the consumption electricity, per capita consumption in India is far below that in other countries. Moreover access to electricity is uneven. In 2003, about 68% of Indian households were connected to the electricity grid with household electricity consumption accounting for about 35% of total electricity consumption, and household and industrial sector consumption combined accounting for 65% of the total 6.209 GWH.[1] Further, details about the institutional background of the Indian electricity supply industry may be found in [2].

Various multi-fractal formalism have recently been developed to describe the statistical properties of the singular measure of signals in terms of their singularity spectrum $f(\infty)$, or generalized dimension D_q . The idea of distribution of singularities all lying on interwoven sets of varying fractal dimensions called multi-fractal was introduced by Frish et al [3]. Further, Hentschel et al [4] introduced the Generalised dimension D_q , and Halsey et al [5] the $f(\infty)$ spectrum. Wavelets are a recently developed concept and WTMM method [6] with continuous basis function, is a well known method to investigate the multi fractal scaling properties of fractal and self affine objects in the presence of non stationaries. In this paper, we intend to clarify the characteristics of electrical power demand using WTMM with continuous function.

II. WTMM METHOD

Finding the distribution of singularities in a multi-fractal signal is particularly important for detecting, identifying, and measuring the scaling behavior. It is possible to measure the singularity spectrum of multi-fractal signals from the wavelet transform local maxima using global partition function. Mallat [7] has proved that all singularities of irregular signal (multi-fractal signal) could be detected using WTMM in partition function and shown the method to measure the local singularity α . Numerical analysis performed in our work was based on WTMM method [6]. This is one of the commonly used approaches to study multi scale structures in complex time series. Using WTMM in calculating the partition function, we can avoid the deviations that are made by the oscillation of wavelet coefficients when $q < 0$. It is more accurate and efficient in detecting singularities in signal. That is, the attractiveness

of using the WTMM is associated with the possibility it provides of analyzing a wide range of scales and a broad spectrum of scaling characteristics from small fluctuations and weak singularities to large fluctuations and strong singularities. In this approach, the numerical quantification of time series is done by the so called singularity spectrum $D(h)$ characterized by the Holder exponent h .

For a continuous process with spectral density $\Gamma_x(\vartheta)$, we define the wavelet transform of a function $f(x)$ as:

$$T\Psi |f(x_0, a)| = \frac{1}{a} \int_{-\infty}^{+\infty} f(x) \Psi\left(\frac{x-x_0}{a}\right) dx \quad (1)$$

Where $f(x_0)$ is a distribution at a point x_0 (location parameter), a is the scale parameter, and Ψ defines the family of wavelets. : that is, for varying values of a , wavelets of different length scales can be constructed. The wavelet transform can be seen as decomposing the function $f(x)$ in to elementary space scale contributions by combining it with a suite of localized functions, the so called wavelets, all of which are constructed by translation and dilation of a single function, the analyzing wavelet. An attractive feature of wavelets is that one could construct various analyzing patterns which satisfy the requirements for a function to be called a wavelet. A local singular behavior of $f(x)$ at the point x_0 results in an increase of $|T\Psi|f(x, a)|$ as $x \rightarrow x_0$, and can be characterized by the Hölder exponent $h(x_0)$ that quantifies the scaling of the wavelet coefficients for small a . In other words, the local singular behavior of f around x_0 is characterized by a power law behavior:

$$T\Psi |f(x_0, a)| \sim a^{T(q)}. \quad (2)$$

Further, the statistical description of local singularities is performed using the notion of the partition function $S(q, a)$, being the sum of the q^{th} powers of the local maxima of $T\Psi|f(x, a)|$ at a scale a . For small a , the partition function $S(q, a)$ scales as:

$$S(q, a) \sim a^{T(q)}. \quad (3)$$

with the scaling exponent $T(q)$. The singularity spectrum $D(h)$ can be estimated using the Legendre transform :

$$D(h) = q(h) - T(q), \quad (4)$$

For positive values of q , the partition function $S(q, a)$ characterize the scaling of large fluctuations in the data (strong singularities). For negative values of q , it reflects the weak singularities. Applications of the WTMM method, to time series allows us to characterize correlations of different types if $h \neq 0$ and $D(h) \neq 0$. In particular, the range $0 < h < 0.5$ implies the presence of anti-correlated behavior, while $h > 0.5$ reflects correlated dynamics.

III. APPLICATION OF WTMM METHOD TO ELECTRIC DEMAND TIME SERIES:

Application of WTMM method to an analysis of point processes extracted from the time series (Fig.1(a) & (b)) of two completely different places: India (Data of electrical power demand obtained at the website: <http://www.cea.nic.in/> statistics.) and China with 8832 points has revealed differences in the long range

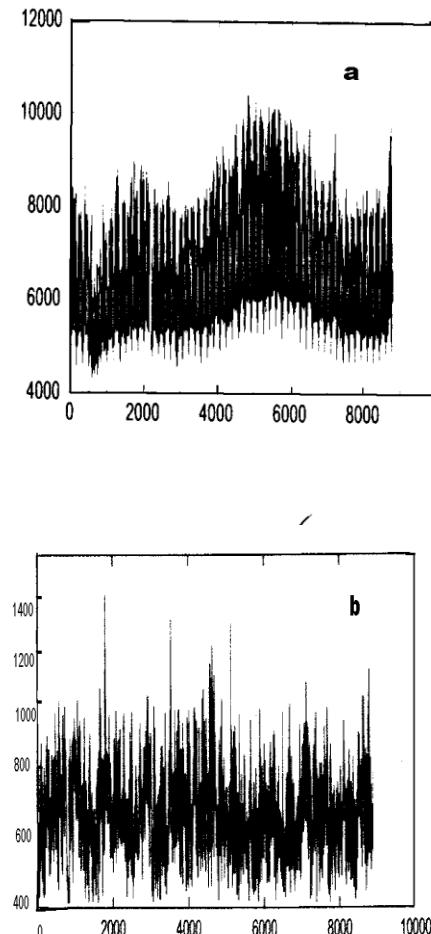


Figure 1. Electrical Power demand Time series for:
(a): China (b) India.

correlations, revealing that power generating system of china is better suited to satisfy correlations in the demand. In the analysis, we use the successive derivatives of a Gaussian function (i.e., the fifth derivative of Gaussian function was chosen) as analyzing wavelet:

$$\Psi^5(t) = d^5 (e^{t^{2/2}}) / dt^5; \quad (5)$$

Twelve wavelet transform data files were obtained applying the wavelet transform to both electrical demand data with

Ψ^5 , ranging the scaling factor α from $\alpha_{\min} = \frac{1}{256}$ to $\alpha_{\max} = 8$ in steps of a power of two. The computed partition function for each one in the range $-20 \leq q \leq 20$ displays nonlinearity, indicating multi-fractality (Figs.2(a) & (b)).

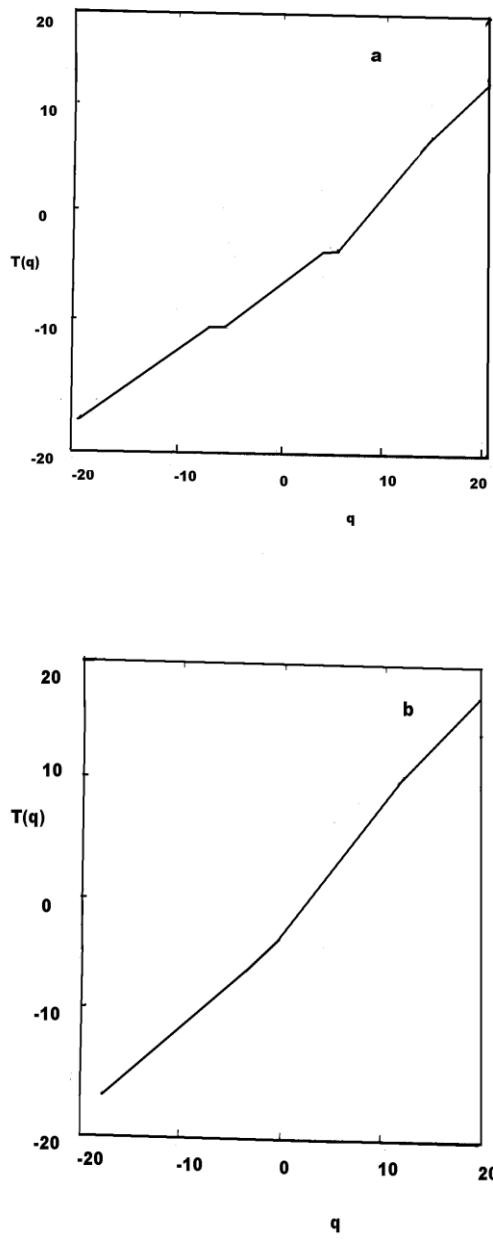
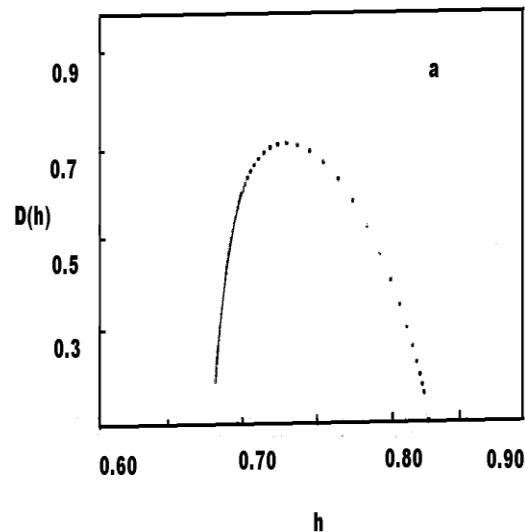


Fig.2. $\tau(q)$ for Electrical power demand of:
(a) China & (b) India.

It is seen that $\tau(q)$ is a nonlinear convex increasing function with $\tau(0) = -0.65$ and the slopes $\alpha_{\min} = 0.66$ for $q \leq 0$ and $\alpha_{\max} = 0.84$ for China, while for India $\tau(0) = -0.54$ and $\alpha_{\min} = 0.62$ for $q \leq 0$ and $\alpha_{\max} = 0.78$. The corresponding singularity spectra $D(h)$, a single humped function with a non unique Holder exponent, are obtained by Legendre transforming $\tau(q)$ (Fig.3.(a) & (b)). As expected from $\tau(q)$,

the support of the $D(h)$ extends over a finite interval whose bounds are $\alpha_{\min} = 0.66$ and $\alpha_{\max} = 0.84$ for China, which is larger than the one for India ranging from $\alpha_{\min} = 0.62$ and $\alpha_{\max} = 0.78$. The minimum value α_{\min} , corresponds to the strongest singularity characterizing the most rarefied zones, while higher values exhibit weaker singularities until α_{\max} which corresponds to the densest zone. α_{\max} & α_{\min} both between 0.5 & 1 signifies persistent processes, (which obeys to the "**Joseph's effect**" in bible of 7 years of loom, happiness & health and 7 years of hungry and illness), although a very little less persistence for the case of India than China (highly persistent) due to slight shift of the curve to the right. However, the analyzed data remain multi-fractal at all stages.

To investigate the underlying evidence for multi-scaling more carefully, it is necessary to present the statistical findings, for both countries. The support dimensions $D_0 = D_{\max} = \tau(0)$ are 0.65 & 0.54 for China & India respectively, implying capacities of the support measure fractional (i.e., chaotic). The numerical values of the holder exponent for the dimension supports, $\alpha(D_{\max})$, are 0.7 for China and 0.69 for India (corresponding to $D(h)_{\max}$), implying the fact that the events with $\alpha = \alpha(D_{\max})$ are the most frequent ones. A Hurst exponent of 0.7 and 0.69 describes a very persistent time series and if the distribution is homogenous, there is a unique h (or α), but if it is not, there are several exponents h (or α), with the most frequent h will characterize the series that will play as Hurst exponent.



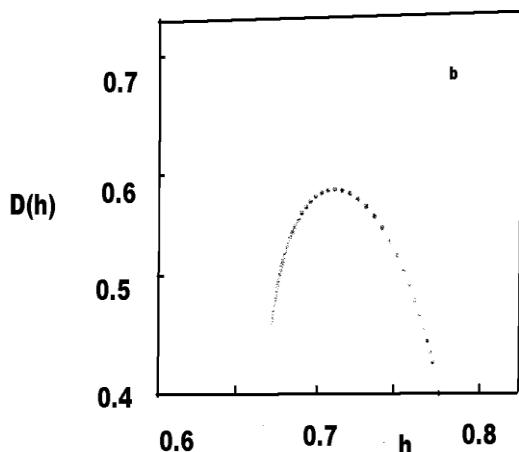


Fig.3. $D(h)$ versus h for electrical demand for:
(a) China and (b) India.

Moreover,, $h^t = (h_{\min} + h_{\max}) / 2$ is almost the same for China (0.75) and India (0.74). This implies that the curves are slightly humped to the left, an effect that is more pronounced for India than for China and a better precision is obtained for the $h > 0$ branch, where the bigger values will prevail (i.e., higher changes in demand of electricity, a rare situation). Further, the asymmetrical shape of the spectrum reveals more pronounced in homogeneities in the events associated with the $q < h < 0$ branch ,associated with smaller values of the power.(Slight change in demand more common).This is indicated by $h_{\text{range}} = (h_{\max} - h_{\min})$ with the case of larger than the other one. The information dimension, on the other hand for china is $D_1 = h(q(1)) = 0.7$ = 0.7 for which features the scaling behavior of the Information while it is $D_1 = h(q(1)) = h$ (0.64) for India.(i.e fractional value for both countries) .That is, for both china and India, the electricity demand corresponds to chaotic systems with the problems of forecasting associated with them.

The correlation dimension are $D_2 = \tau(2) = 0.79$ for India and 0.87, characterizing chaotic attractor, with $D_2 > 1/2$ indicates long range correlations, with the characteristic behavior of adaptively that plays the role of organizing principle for highly complex, non linear processes that generates fluctuations on a wide range of time scales and ,in addition, the lack of characteristic scale helps in preventing excessive mode locking that would restrict the reaction of the event. In short, we can see that, there is longer range correlation for china, implying that for the case of an abrupt change of the demand china electrical system will have a better answer.

V. CONCLUSION.

We found that both the electrical demand behave, like most ones in nature, as long term memory phenomena. in both

cases the fractal dimension obtained correspond to chaotic processes. Moreover, the correlation dimension gives us an account of long range correlation, with the china long range correlated than India. This lays that china power system is better suited to satisfy oscillations in the demand .in spite of changes, of h ranges within $0.5 < h < 1$, the greater value for India indicates that the demand series varies in wider ranges, which features the variation on demography.

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